Complex Systems Presentation

Fern Y. Hunt
MCSD/ITL
March 17, 2008

Modelling Network Control-an optimization approach

Customers (or users) who want to transmit from one location to another in the network are given a certain amount of bandwidth from a source

Each link in the transmission path has capacity

c_l $\quad l \in L(s)$ set of links used by s

Source decision: How much traffic to send over the links?
(TCP)

TCP-AQM Modelling

Through a process of reverse engineering a number of researchers came to interpret TCP congestion control as carrying out a <u>distributed</u> primal dual alogrithm to maximize aggregate utility.

GOAL OF A CONTROL ALGORITHM: optimize a global measure of network performance

Utility Functions

Routing

Wang, Li, Low and Doyle (WLLD) ’05

Investigated cross layer utility maximization allowing route selection.
Suppose multiple paths are available for a single source-destination.
If minimum cost routing is selected for each source-destination pair this is OSPF (open shortest path first) protocol.

Source’s decisions
•Congestion control (how much bandwidth to give)
•How to distribute traffic over available paths in order to maximize aggregate utility

Optimization Problem

$R_{ls} = r_{ls} > 0$ iff link l is on a route used by s

$= 0$ otherwise

WLLD analyze the case $r_{ls} = 1$ and propose an algorithm for its solution that models TCP-IP.

$r_{ls} = 1$ no splitting of traffic among routes; equilibrium for algorithm iff there is no duality gap.

Duality gap exists for two links topology; route flapping, reduction in utility

If $r_{ls} < 1$ splitting among routes allowed, no duality gap. WLLD claim higher utility is obtained when TCP-IP has no equilibrium.

r_{ls} =1 is single path utility maximization not minimum cost routing.

Random Assignment of Routes

Marbukh and Klink ’07 proposed a scheme for achieving aggregate utility maximization by randomly allocating traffic according to a route probability distribution.

By controlling the degree of randomness, simulations of the scheme showed:

•Using a minimum average link cost rather than an absolute minimum cost can enhance network performance.

•There is a tradeoff between stability and performance. Maximum performance occurs near the border between regions of stability and instability– route flapping phenomenon

Our Results

We reformulated this scheme so that it fits into the framework of L&L and WLLD. This will allow us to do a systematic theoretical study of how such a protocol might behave because we reduce the problem to an investigation of the dynamics of a primal dual algorithm

PRELIMINARY RESULTS:

•Investigated two simple topologies twolinks and diamond where we confirmed the stability trade-offs discussed in Marbukh/Klink

•Study suggests a new control strategy of switching route allocations without changing the randomness parameter h_s.

•Dynamical behaviour not generally discussed in the literature

Problem Formulation

s – a source/destination pair

x_s – bandwidth assigned to s $R_s(l)$ = set of paths or routes for s that use link l.

Traffic for source s is assigned to route $r \in R_s(l)$ with probability β_{sr}.

We require that $\beta_{sr} > 0$, for $r \in R_s(l)$. Let $\hat{\beta} = \{\beta_{sr}\}$.

The optimization problem that our protocol seeks to solve is:

$$\max_{\hat{x} \geq 0} \max_{\hat{\beta} \geq 0} \sum_s U_s(x_s)$$

$$\sum_s \sum_{r \in R_s(l)} \beta_{sr} x_s \leq c_l$$

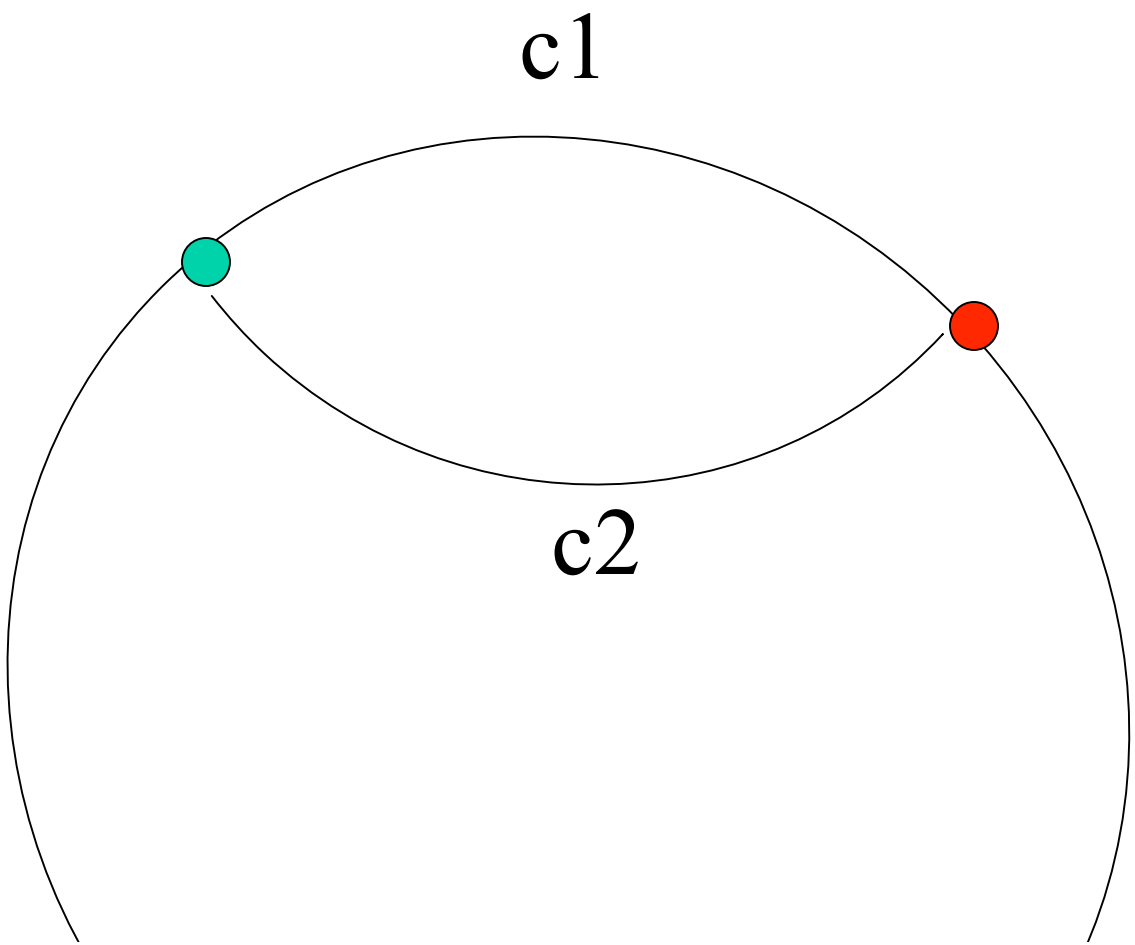
$$\forall s \qquad \sum_{r \in R_s} \beta_{sr} = 1 \qquad R_s = \text{set of routes for } s$$

$$-\sum_{r \in R_s} \beta_{sr} \log \beta_{sr} \geq h_s$$

Problem Formulation (cont'd)

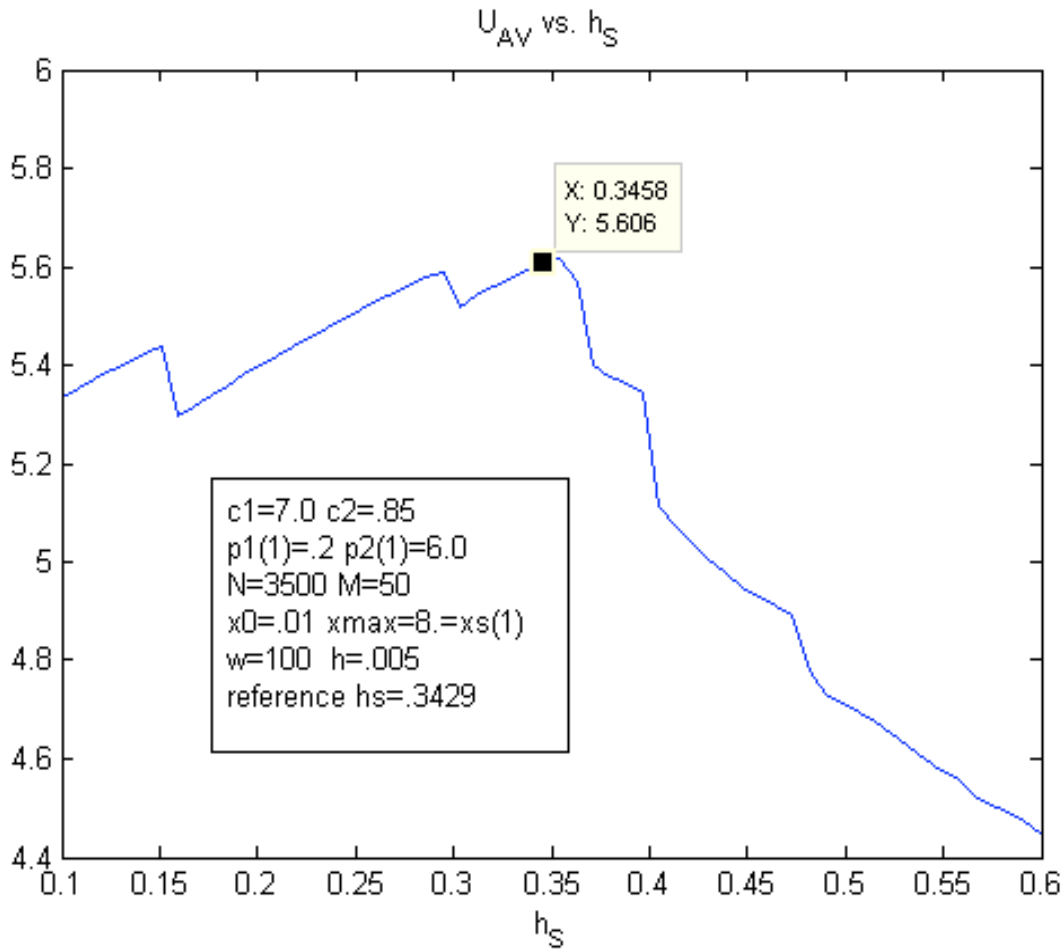
The problem on the previous slide is not convex. We introduced a change of variables so that the resulting problem is convex and there is no duality gap.
A Lagrangian formulation of this problem allows us introduce link prices as in L&L and WLLD.
The algorithm for solving the Lagrangian form of our problem comes from a gradient projection scheme .
In the case of convergence to an equilibrium steady state is a solution of our original optimization problem.

Two Links Route



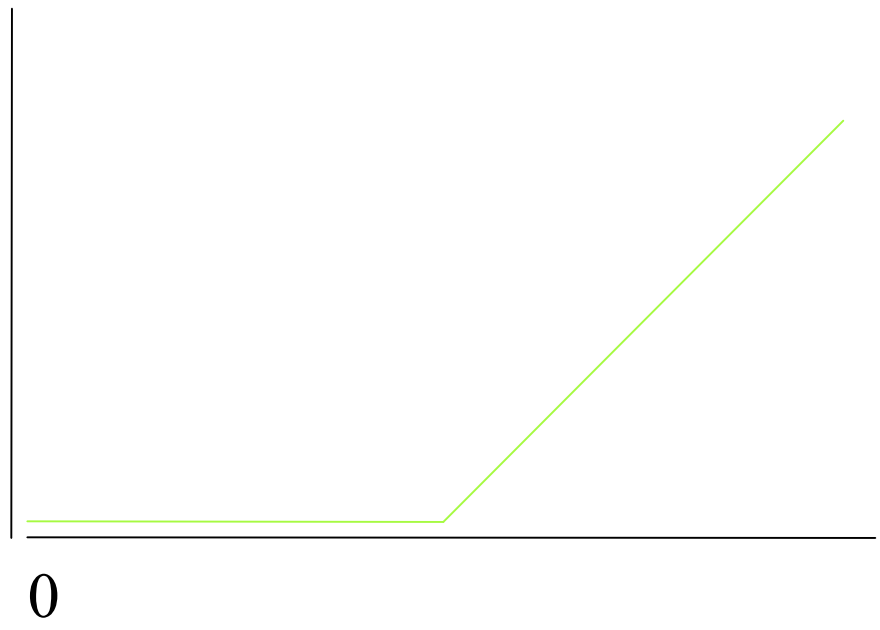
Two Links Example

Two Links Performance Measure



Some Details of Dynamics

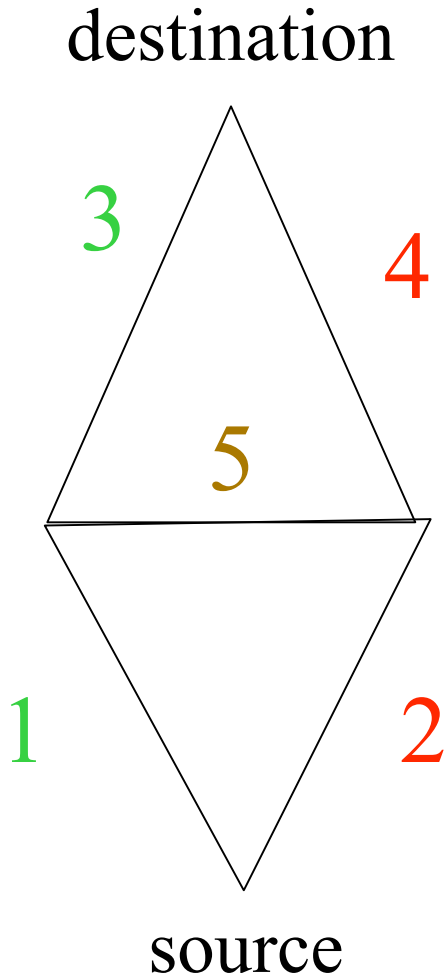
Unconventional Dynamical System

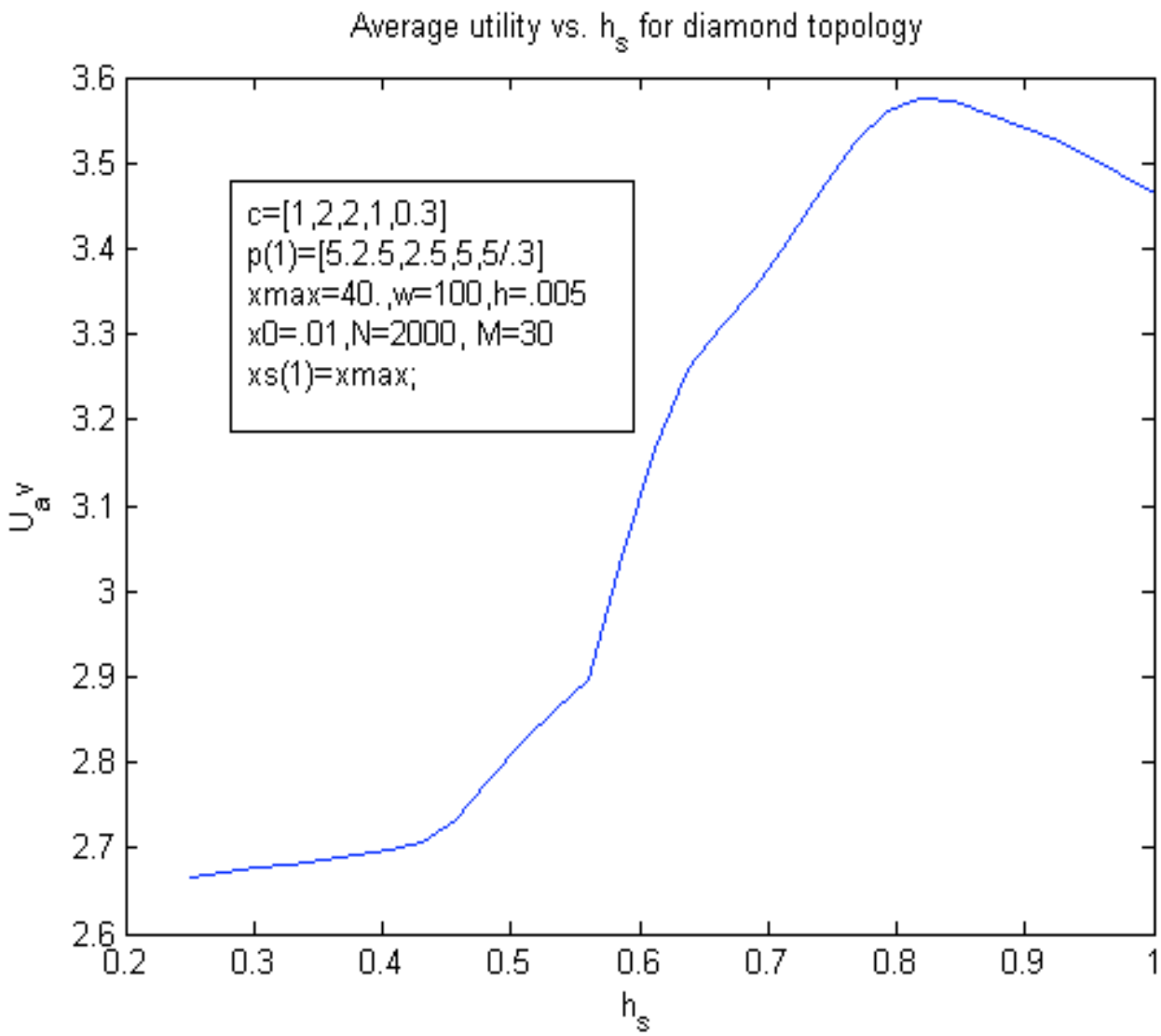


Non-differentiable RHS

Unconventional Dynamical System (cont'd)

DIAMOND



Route	Links
1	1,3
2	2,4
3	1,5,4

Diamond Network Performance vs. hs