

Approximations to link reliability for combined lognormal shadowing and Rayleigh fading

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30 August 2002

In a mobile radio communication system, the signal strength at the receiver may be subject to lognormal fading, Rayleigh fading, or both [1]. Lognormal fading, for which the signal strength in dB (decibel) units is a Gaussian random variable, usually is associated with the uncertainty and/or variation of path propagation loss as the mobile changes location—into or out of “shadow” zones. Rayleigh fading describes the statistical variation in the envelope of the received (continuous wave (CW)) signal when it is the result of the superposition of many versions of the wave that have been reflected from different points.

When the signal power is subject to lognormal fading, the probability density function (pdf) for the received signal power in dBm (dB relative to 1 mW), denoted S , is given by

$$p_{SL}(\xi) = \frac{1}{\sigma_{dB} \sqrt{2\pi}} \exp \left\{ -\frac{(\xi - S_{med})^2}{2\sigma_{dB}^2} \right\} \quad (1)$$

where σ_{dB}^2 is the variance of the signal power in dBm and S_{med} is its median value in dBm (also its average dBm value). That is, the signal power in dBm is a Gaussian random variable. When the signal is subject to Rayleigh fading, its power (not in dB units), denoted P , is exponential distributed and has the pdf given by

$$p_P(\eta) = \frac{1}{P_{avg}} e^{-\eta/P_{avg}} \quad (2)$$

The reliability of the link under Rayleigh fading, denoted p , is the probability that the signal power exceeds some required value in dBm, S_{req} , and is given by

$$p = \Pr \left\{ P > 10^{S_{req}/10} \right\} = \exp \left\{ -\frac{10^{S_{req}/10}}{P_{avg}} \right\} \quad (3)$$

If we use the Suzuki model [2, 3] and equate the average signal power assumed in (2), when expressed in dBm, with the lognormally fluctuating power having the pdf in (1), that is,

$$P_{avg} = 10^{S/10} \quad \text{or} \quad S = 10 \cdot \log_{10} P_{avg} \quad (4)$$

then the link reliability has the value

$$p = E_S \left\{ \Pr \left\{ P > P_{avg} \mid P_{avg} = S \right\} \right\} = E_S \left\{ \exp \left\{ -10^{(S_{req}-S)/10} \right\} \right\}$$

$$= \int_{-\infty}^{\infty} \frac{d\xi}{\sigma_{dB} \sqrt{2\pi}} \exp \left\{ -\frac{(\xi - S_{med})^2}{2\sigma_{dB}^2} - 10^{(S_{req}-\xi)/10} \right\} \quad (5a)$$

$$= \int_{-\infty}^{\infty} \frac{d\xi}{\sigma_{dB} \sqrt{2\pi}} \exp \left\{ -\frac{\xi^2}{2\sigma_{dB}^2} - \frac{10^{-\xi/10}}{M} \right\} \quad (5b)$$

where $M = 10^{M_{dB}/10}$ is the link margin and $M_{dB} = S_{med} - S_{req}$ is the link margin in dB units. The form of (5a) is similar to that for the same quantity in [4].

A simpler form for the reliability p is found by reversing the order of integration—that is, by backing up one step from (3) to take the lognormal shadowing probability as the conditional probability and average over the Rayleigh power fading factor:

$$p = \Pr \left\{ P > 10^{S_{req}/10} \right\} = \Pr \left\{ b \cdot 10^{(\sigma_{dB}G + S_{med})/10} > 10^{S_{req}/10} \right\} \quad (6)$$

where b is the exponentially distributed fading factor with mean value equal to 1 and G is a zero-mean, unit-variance Gaussian random variable.. Using $\beta = (\ln 10)/10$, the reliability can be evaluated as

$$p = E_b \left\{ G > -\frac{M_{dB}}{\sigma_{dB}} - \frac{\ln \eta}{\beta \sigma_{dB}} \mid b = \eta \right\} = E_b \left\{ P_G \left(\frac{M_{dB}}{\sigma_{dB}} + \frac{\ln b}{\beta \sigma_{dB}} \right) \right\}$$

$$= \int_0^{\infty} d\eta e^{-\eta} P_G \left(\frac{M_{dB}}{\sigma_{dB}} + \frac{\ln \eta}{\beta \sigma_{dB}} \right) \quad (7a)$$

$$= \int_0^1 d\eta P_G \left(\frac{M_{dB}}{\sigma_{dB}} + \frac{\ln(-\ln \eta)}{\beta \sigma_{dB}} \right) \quad (7b)$$

where $P_G(x) = \Pr \{ G \leq x \}$ is the Gaussian cumulative probability distribution function (cdf). The simple integrand and finite interval of integration in (7b) especially facilitate accurate numerical integration. Example results for the link reliability p are shown in Figure 1.

The similarity of the curves in Figure 1 for combined shadowing and fading to those for shadowing only suggests a functional form for p that is based on the Gaussian cdf. With some calculation, the following new “closed form” approximation for the link reliability was found that has better than 1-dB accuracy for reasonable values of the reliability:

$$p \approx e^{-e^{-\beta(M_{dB}+6)}} P_G \left(\frac{M_{dB}}{\sigma_{dB}} e^{-3/\sigma_{dB}} \right) \quad (8)$$

Values of this expression are compared with computations of the reliability in Figure 2. The inverse of this expression has the following approximation, also with about 1-dB accuracy:

$$M_{dB} \approx \sigma_{dB} e^{-(\sigma_{dB}^{-2})/4} \left[P_G^{-1}(p) + 0.35 - 0.015\sigma_{dB} \right] \quad (9)$$

Values of this expression are compared with computations of the margin in Figure 3. These results are useful for predicting the margin needed in radio link budget analysis, and provide a more accurate method than suggested in [1], which is to calculate margins separately for shadowing alone and fading alone and combine the two margins. As Figure 1 shows, the actual margin in dB that is needed when both shadowing and fading are happening is less than the sum of the two separate margins.

References

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- 3 H. Suzuki, “A Statistical Model for Urban Radio Propagation,” *IEEE Trans. on Comm.*, vol. COM-25, pp. 673-680 (July 1977).
- 4 K. Siwiak, “Impact of Ultra Wide Band Transmissions on a Generic Receiver,” *Proc. VTC 2001*, pp. 1181-1183.

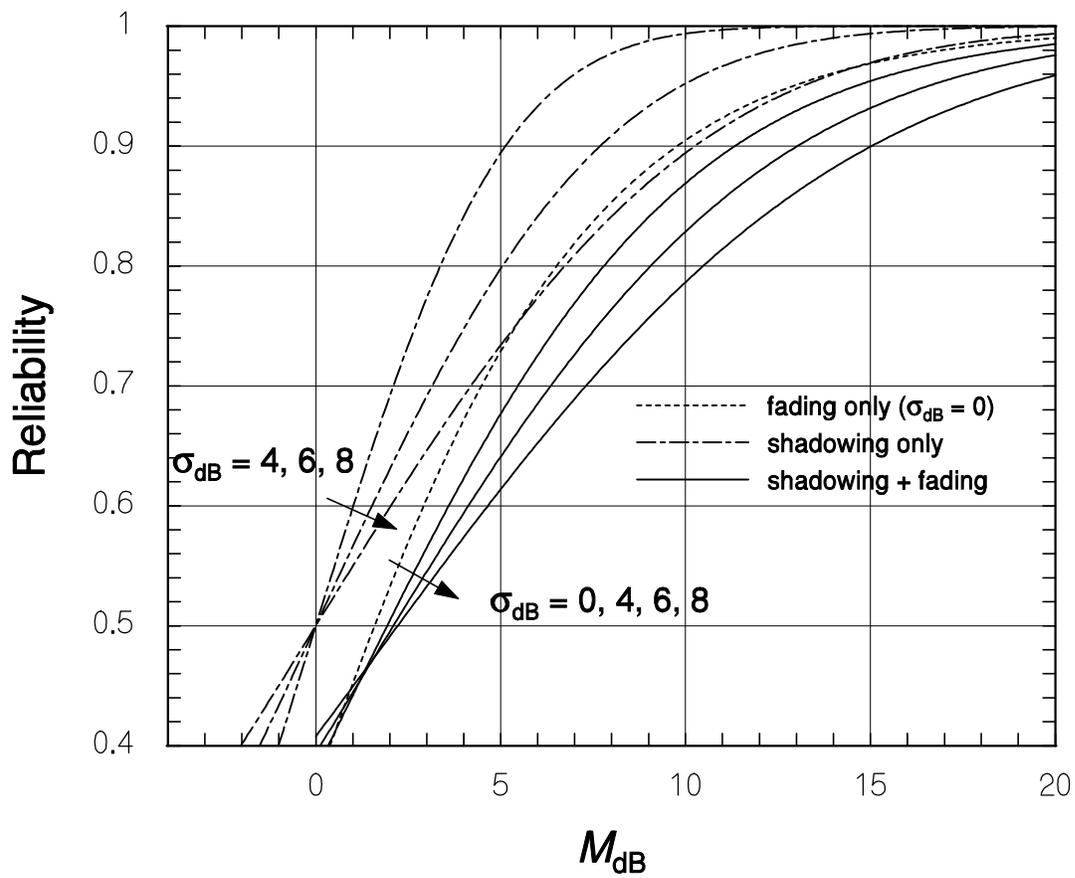


Figure 1. Calculations of link reliability as a function of margin in dB.

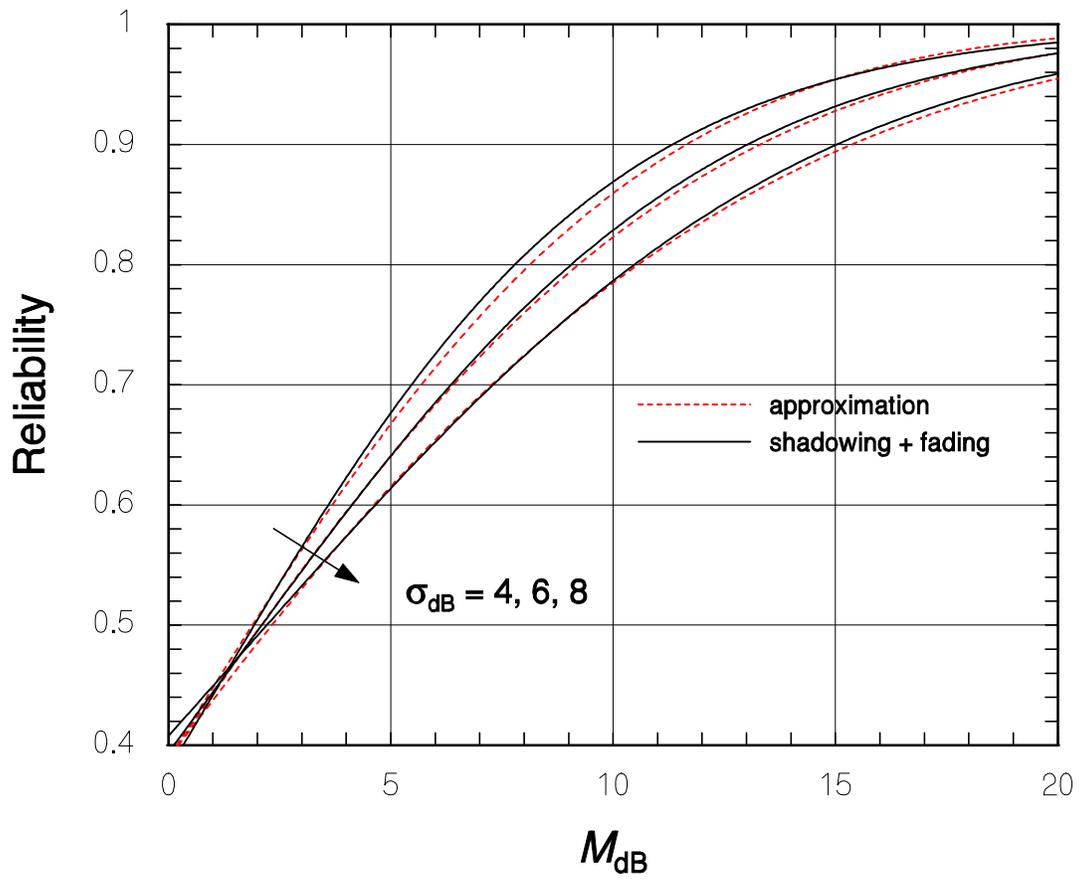


Figure 2. Computed and approximate values of link reliability for shadowing and fading.

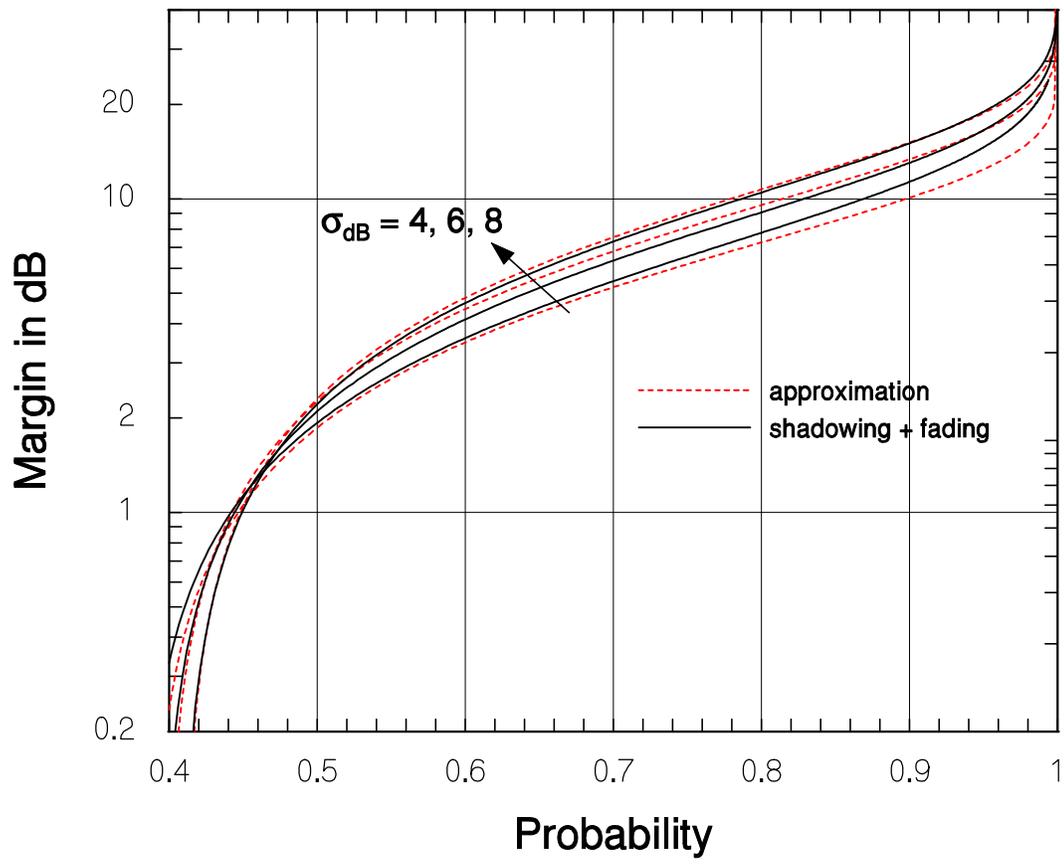


Figure 3. Computed and approximate link margins for shadowing and fading.