

CENTER FOR APPLIED MATHEMATICS SEMINAR

Techniques for the Fitting and Verification
of Linear/Non-Linear Models using DATAPLOT

James J. Filliben*
National Bureau of Standards
Washington, D. C. 20234

Green Auditorium, Administration Building
10:30--12:00, Monday, November 20, 1978

ABSTRACT

An important practical problem which arises in a wide variety of NBS experimental activities is that of model-building and verification. Heretofore, linear models have always played (due to their simplicity and mathematical tractability) an important role in describing physical phenomena and in gaining insight into underlying mechanisms. Such insight has frequently resulted, however, in a growing awareness of the fundamental limitations of linear models and a begrudging acceptance of the fact that many phenomena are intrinsically non-linear.

The main purpose of this talk is to convey the fact that recent advances in statistical software have been made with the net effect that the non-linear modeling problem is now "solved" in the sense that fitting such models is now no longer a major programming effort for the analyst--rather, it has become a one line/one command operation. The use of the DATAPLOT FIT command frees the analyst from typical programming details, and allows the analyst to concentrate on the physical modeling problem at hand. Whereas before, the several iterations typical for exploratory non-linear model-building would take weeks and months, it now takes minutes. Enormous savings of time--the analyst's time--thus result.

This talk will demonstrate by several examples the ease with which the analyst may now carry out non-linear (or linear) modeling at NBS. In addition to providing details regarding the DATAPLOT FIT command (by which such non-linear modeling is done), this talk will also cover:

- 1) general principles for non-linear model construction;
- 2) guidelines for which models to choose (and avoid);
- 3) techniques for model verification;
- 4) considerations for choosing between competing models;
- 5) statistical and graphical techniques for assessing goodness of fit.

An indefinitely large number of non-linear models can be handled with the DATAPLOT FIT command; among the more popular and frequently-encountered models that will be explicitly included in the discussion and examples are the following:

- 1) linear and polynomial models;
- 2) exponential models;
- 3) models involving powers to be estimated;
- 4) square root models;
- 5) exponential over polynomial models;
- 6) Lorentzian models;
- 7) Gaussian models;
- 8) Bessel function models;
- 9) Chebychev function models;
- 10) rational function models.

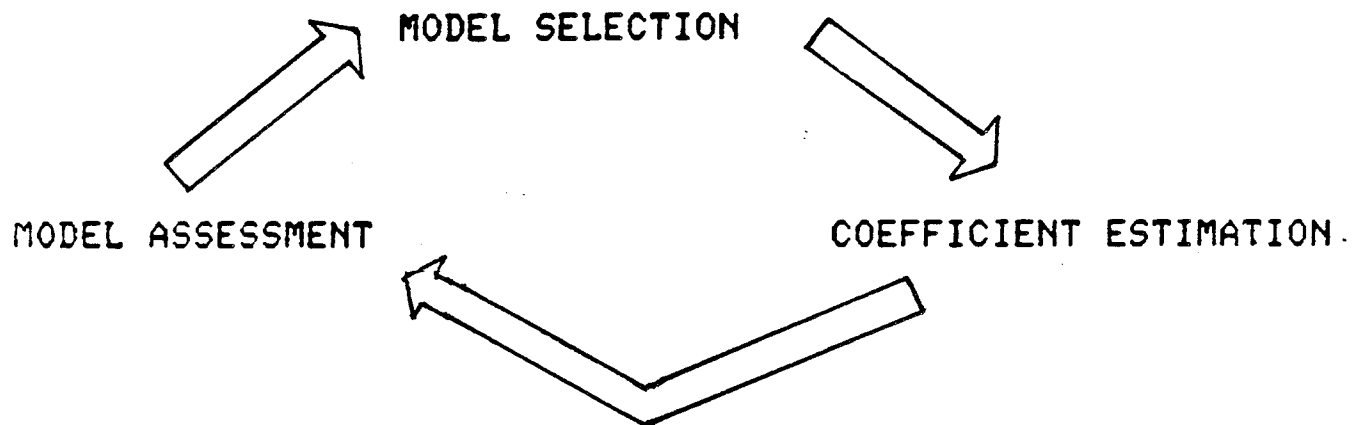
The importance of this last family (rational functions) as a general and flexible modeling family will be discussed and demonstrated. The handout from this talk will illustrate the application of the DATAPLOT FIT command to over 25 different NBS linear/non-linear modeling problems; DATAPLOT instructions and corresponding output will be provided for each example.

EXAMPLES

| | |
|---|-------------------------------------|
| 1. ALASKA PIPELINE RADIOGRAPHIC DEFECT BIAS CURVE | LINEAR |
| 2. CONCRETE TENSILE STRENGTH STUDY | LINEAR |
| 3. FIRE RESEARCH IGNITION STUDY | LINEAR |
| 4. OIL CONSUMPTION STUDY | LINEAR |
| 5. NUCLEAR SAFEGUARDS TANK CALIBRATION CURVE | QUADRATIC |
| 6. SILICON 288.1 NM. CALIBRATION CURVE | LINEAR & QUADRATIC |
| 7. LOAD CELL CALIBRATION CURVE | |
| 8. COMPUTER UTILIZATION STUDY | EXPONENTIAL |
| 9. SAFETY EQUIPMENT STUDY | EXPONENTIAL |
| 10. ERYTHEMA BIOMEDICAL STUDY | 2 EXPONENTIALS |
| 11. QUEUEING THEORY SERVER FUNCTION CURVE | POWER |
| 12. CONCRETE STRENGTH STUDY | POWER IN DENOMINATOR |
| 13. DENTAL RESEARCH MONOMOLECULAR ADSORPTION STUDY | EXP., QUAD., ., RECIPROCAL |
| 14. QUANTUM DEFECTS FOR SULFUR I ATOM STUDY | ARCTANGENT |
| 15. CIRCULAR INTERFERENCE FILTER STUDY | LORENTZIAN & GAUSSIAN |
| 16. ULTRASONIC CALIBRATION CURVE | BESSEL |
| 17. ULTRASONIC REFERENCE BLOCK ANALYSIS | EXPONENTIAL/LINEAR |
| 18. CONCRETE PULL-OUT BOND STRENGTH STUDY | LINEAR/LINEAR |
| 19. SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS | LINEAR/LINEAR & QUADRATIC/QUADRATIC |
| 20. SEMICONDUCTOR BORON DIFFUSION STUDY | QUADRATIC/QUADRATIC |
| 21. COPPER THERMAL EXPANSION CURVE | QUADRATIC/QUADRATIC & CUBIC/CUBIC |
| 22. SMOKE OBSCURATION FIRE RESEARCH STUDY | LINEAR/QUADRATIC & LINEAR/QUARTIC |
| 23. DOPPLER SPECTROMETER PARTICLE SIZE DIST. STUDY | LINEAR/QUADRATIC |
| 24. RESIDENTIAL TIME-TEMPERATURE CURVE | CUBIC/CUBIC |
| 25. PRACTICAL TEMPERATURE SCALE REFERENCE CURVE | CUBIC/CUBIC |
| 26. PHOSPHORUS-DOPED SILICON SEMICONDUCTOR STUDY | CUBIC/CUBIC |
| EOF AT LINE 35 | |
| 35*) | |

MODEL BUILDING IS AN ITERATIVE PROCESS

PLOTTING IS A NECESSARY COMPONENT FOR
MODEL SELECTION AND VERIFICATION



MODEL ASSESSMENT IS PRIMARILY DONE BY PREDICTED VALUE AND
RESIDUAL ANALYSIS

- 1) RESIDUAL STANDARD DEVIATION
- 2) RESIDUAL PLOTS

3) PREDICTED VALUE SUPERIMPOSED ON RAW DATA PLOT

@DATAPLOT

END <ANALYST PROGRAM OR INTERACTIVE SESSION (OR BOTH)>
(OR STOP OR HALT OR EXIT)

ELEMENTS OF THE DATAPLOT LANGUAGE

AND
 ANOVA
 AMPLITUDE PLOT
 ARGAND PLOT
 AUTOCORRELATION PLOT
 BATCH
 BELL
 CALCOMP
 CHARACTERS
 COHERENCY SPECTRUM
 COMMENT
 COMPLEX DEMODULATION ... PLOT
 CONTINUOUS TERMINAL
 ... CONTROL CHART
 CO-SPECTRUM
 CROSS-CORRELATION PLOT
 CROSS-SPECTRUM
 CUMULATIVE FREQUENCY PLOT
 CUMULATIVE HISTOGRAM
 DELETE
 DEGREE
 DISCRETE TERMINAL
 ECHO
 END
 ERASE
 EXACT ... RATIONAL FIT
 EXIT
 FIT
 FOR
 FRAME
 FREQUENCY PLOT
 GAIN PLOT
 GRID
 HALT
 HARDCOPY
 HELP
 HISTOGRAM
 LAG-1 AUTOCORRELATION PLOT
 LET
 LOGLOG
 LOGX
 LOGY
 OFF
 ON
 PACK
 PERCENT POINT PLOT
 PHASE PLOT
 PIE CHART
 PLOT
 PRINT
 PRINTER
 ... PROBABILITY PLOT
 QUADRATURE SPECTRUM
 RADIAN
 ... RANDOM NUMBERS
 READ
 RESET
 RUN-SEQUENCE PLOT
 SEQUENCE
 SIZE
 SKIP
 SMOOTH
 SORT
 SPECTRUM
 STATUS
 SUBSET
 TEKTRONIX
 TITLE
 UNFRAME
 UNGRID
 UNLABEL
 UNLOG
 UNSORT
 UNTITLE
 VERSATEC
 VERSUS
 WRITE
 XLABEL
 X1LABEL
 X2LABEL
 X3LABEL
 XLIM
 XMAX
 XMIN
 YLABEL
 YLIM
 YMAX
 YMIN

REFERENCES FOR FITTING CURVES TO DATA
(FUNCTIONAL CONSIDERATIONS)

HASTINGS, C. (1955). APPROXIMATIONS FOR DIGITAL
COMPUTERS. PRINCETON UNIVERSITY PRESS.

HOERL, ARTHUR E. CHAPTER 20 IN THE CHEMICAL
BUSINESS HANDBOOK.

READ DATA

PLOT DATA

FIT DATA

SUPERIMPOSE DATA & PREDICTED VALUES

PLOT RESIDUALS

EXAMPLE 1-A

READ JJF6*DATA.BERGER1 TRUE MEAS
 FIT MEAS = A0 + A1*TRUE

LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 107

MODEL-- MEAS = A0 + A1*TRUE

REPLICATION CASE

REPLICATION STANDARD DEVIATION = .6112686932+01

REPLICATION DEGREES OF FREEDOM = 29

NUMBER OF DISTINCT SUBSETS = 78

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL * STANDARD * DEVIATION * | PARAMETER ESTIMATES |
|---------------------|------------------------|---|------------------------|
| 1-- | .10000-01 | .11069+02 * | .10000+01 .10000+01 |
| 2-- | .50000-02 | .60809+01 * | .49883+01 .73124+00 |

FINAL PARAMETER ESTIMATES

(APPROX. ST. DEV.)

1 A0 4.99369

(1.126)

2 A1 .731111

(.2455-01)

RESIDUAL STANDARD DEVIATION = 6.0809237957

RESIDUAL DEGREES OF FREEDOM = 105

REPLICATION STANDARD DEVIATION = 6.1126869321

REPLICATION DEGREES OF FREEDOM = 29

LACK OF FIT F RATIO = .9857 = THE 46.3056% POINT OF THE
 F DISTRIBUTION WITH 76 AND 29 DEGREES OF F EEDOM

EXAMPLE 1-B

COMMENT EXAMPLE--ALASKA PIPELINE RADIOGRAPHIC DEFECT BIAS CURVE
COMMENT MODEL --LINEAR

ECHO ON
HARDCOPY ON
BELL ON

READ JJF6*DATA.BERGER1 TRUE MEAS

CHARACTERS X
LINES
PLOT MEAS TRUE

FIT MEAS = $A_0 + A_1 \times \text{TRUE}$

TITLE ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE

YLABEL MEASURED DEPTH

XLABEL TRUE DEPTH (IN .001 INCH)

X2LABEL MODEL-- $M = A_0 + A_1 \times T$

X3LABEL JJF6*CS9.NONLINEAR37

JJF6*DATA.BERGER1

11/15/78

CHARACTERS X BLANK

LINES BLANK SOLID

PLOT MEAS PRED VS TRUE

YLABEL RESIDUALS
PLOT RES TRUE

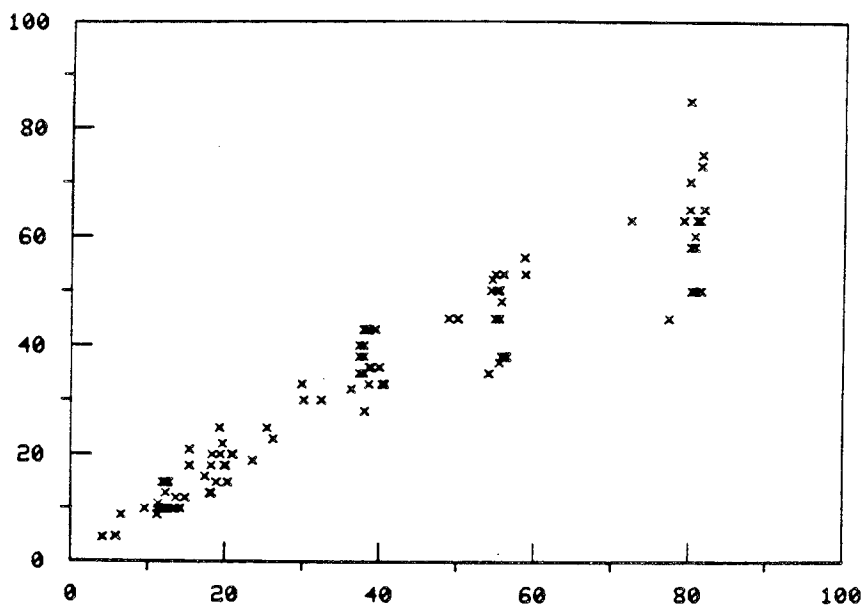
LINES BLANK DASHED

PLOT RES TRUE AND

PLOT Y = 0 FOR X = 0 10 100

XLABEL

NORMAL PROBABILITY PLOT RES



LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 107

MODEL-- MEAS = A0 + A1*TRUE

REPLICATION CASE

REPLICATION STANDARD DEVIATION = .6112686932+01

REPLICATION DEGREES OF FREEDOM = 29

NUMBER OF DISTINCT SUBSETS = 78

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | * * * | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|-------------|------------------------|
| 1-- | .10000-01 | .11069+02 | * | .10000+01 .10000+01 |
| 2-- | .50000-02 | .60809+01 | * | .49883+01 .73124+00 |

FINAL PARAMETER ESTIMATES

(APPROX. ST. DEV.)

1 A0 4.99369

(1.126)

2 A1 .731111

(.2455-01)

RESIDUAL STANDARD DEVIATION = 6.0809237957

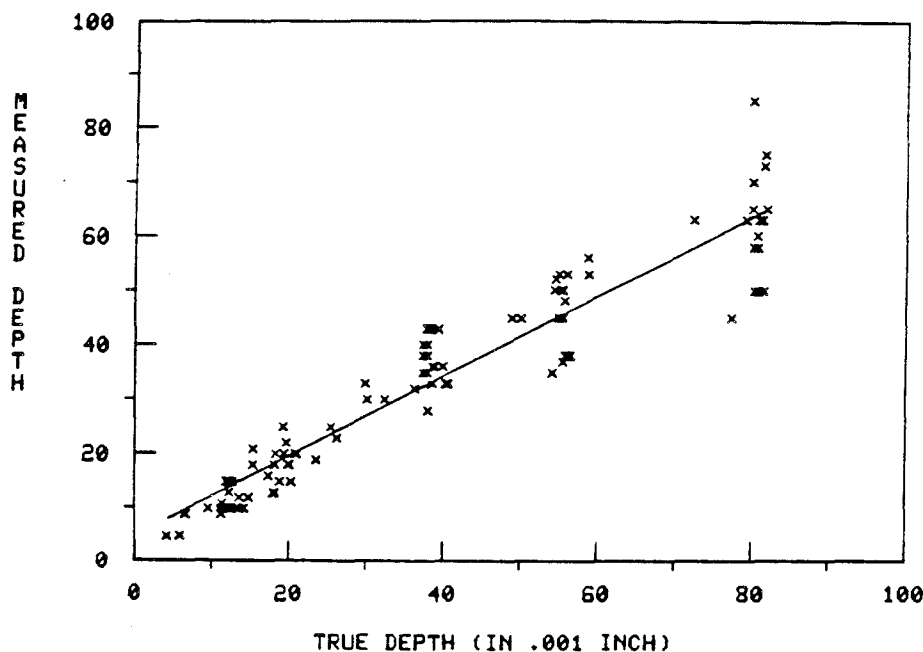
RESIDUAL DEGREES OF FREEDOM = 105

REPLICATION STANDARD DEVIATION = 6.1126869321

REPLICATION DEGREES OF FREEDOM = 29

LACK OF FIT F RATIO = .9857 THE 6.3056% POINT OF THE
F DISTRIBUTION WITH 76 AND 29 DEGREES OF FREEDOM

ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE

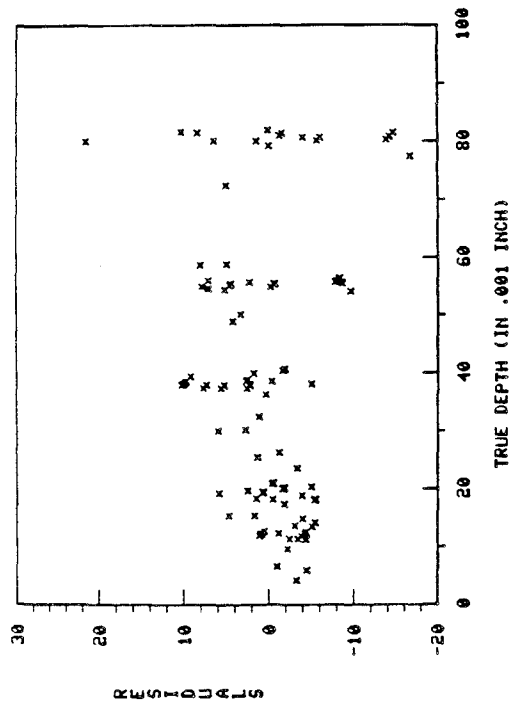


JJF6*CS9.NONLINEAR37

JJF6*DATA.BERGER1

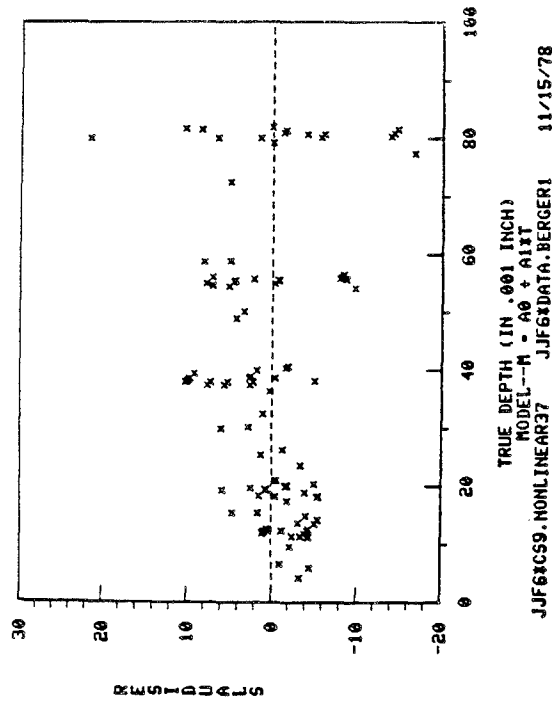
11/15/78

ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE



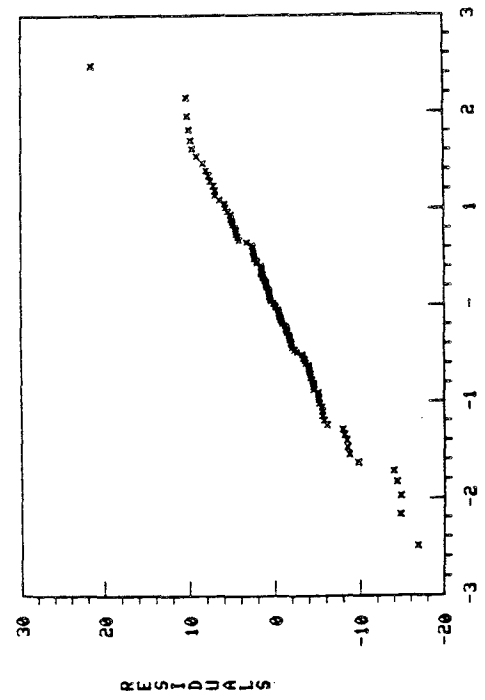
TRUE DEPTH (IN .001 INCH)
MODEL--M = A0 + A1X
JJF61CS9.NONLINEAR37 JJF61DATA.BERGER1 11/15/78

ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE



TRUE DEPTH (IN .001 INCH)
MODEL--M = A0 + A1X
JJF61CS9.NONLINEAR37 JJF61DATA.BERGER1 11/15/78

ALASKA PIPELINE RADIOGRAPHIC DEFECT DEPTH BIAS CURVE



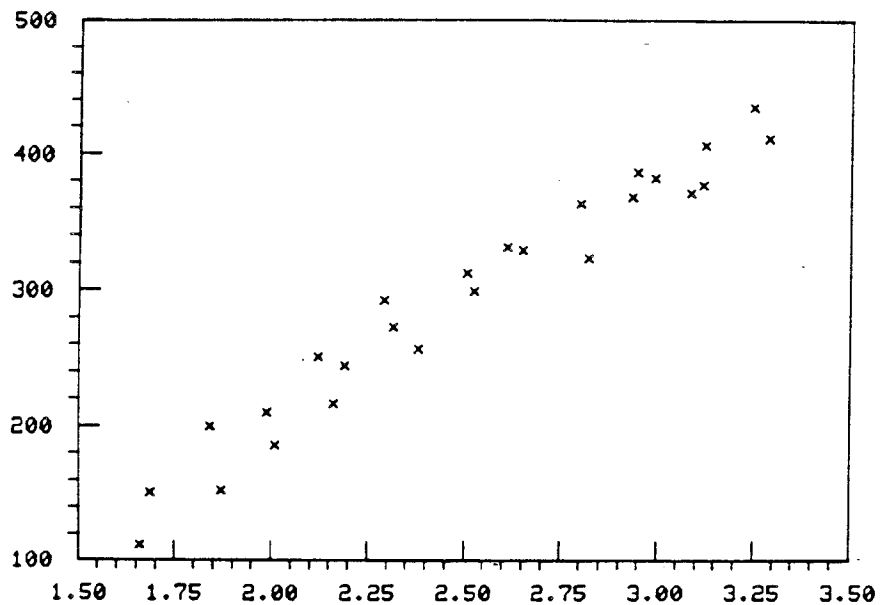
MODEL--M = A0 + A1X
JJF61CS9.NONLINEAR37 JJF61DATA.BERGER1 11/15/78

EXAMPLE 2

```

COMMENT EXAMPLE--H. S. LEW CONCRETE TENSILE STRENGTH
COMMENT MODEL  --LINEAR
.
ECHO ON
HARDCOPY ON
BELL ON
.
SKIP 10
READ JJF6*DATA.LEW3 STREN MAT
.
CHARACTERS X
LINES
PLOT STREN MAT
.
FIT STREN = C0 + C1*MAT
.
TITLE SPLITTING TENSILE STRENGTH OF CONCRETE
YLABEL TENSILE STRENGTH
XLABEL MATURITY (= [TEMPERATURE(F) - 10] * ELAPSED DAYS)
X2LABEL MODEL--STREN = C0 + C1*MAT
X3LABEL JJF6*CS9.NONLINEAR38 JJF6*DATA.LEW3 1/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT STREN PRED VS MAT
.
YLABEL RESIDUALS
PLOT RES MAT
.
XLABEL
NORMAL PROBABILITY PLOT RES

```



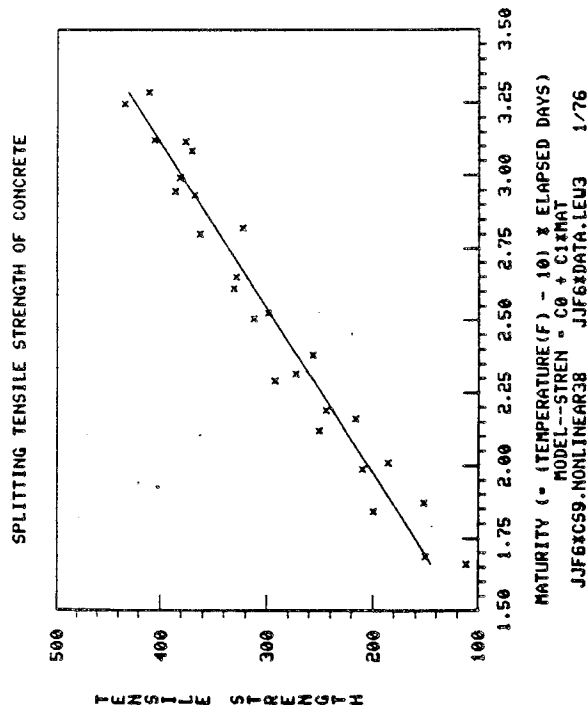
LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 26
 MODEL--STREN = C0 + C1XMAT
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .31561+03 | .10000+01 |
| 2-- | .50000-02 | .19545+02 | -.14376+03 |
| 3-- | .25000-02 | .19542+02 | -.14532+03 |

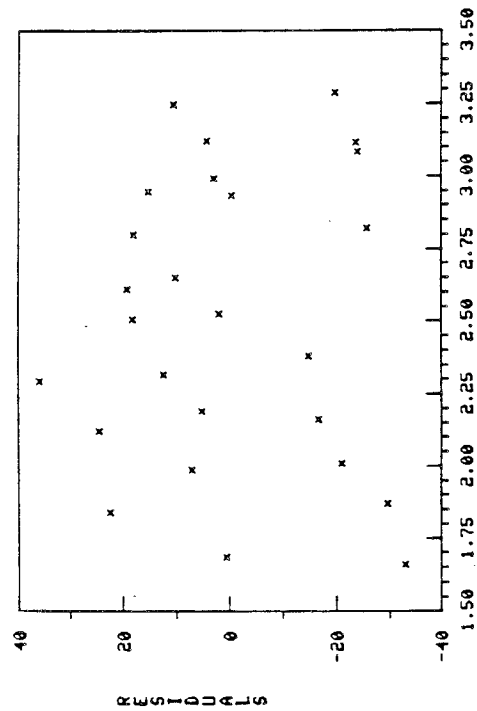
.10000+01
 .17456+03
 .17518+03

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 C0 -145.324 (19.93)
 2 C1 175.178 (7.804)

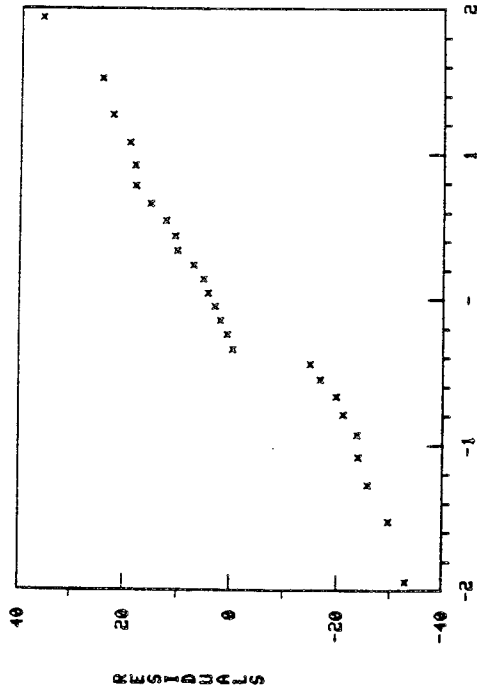
RESIDUAL STANDARD DEVIATION = 19.5424284935
 DEGREES OF FREEDOM = 24



SPLITTING TENSILE STRENGTH OF CONCRETE

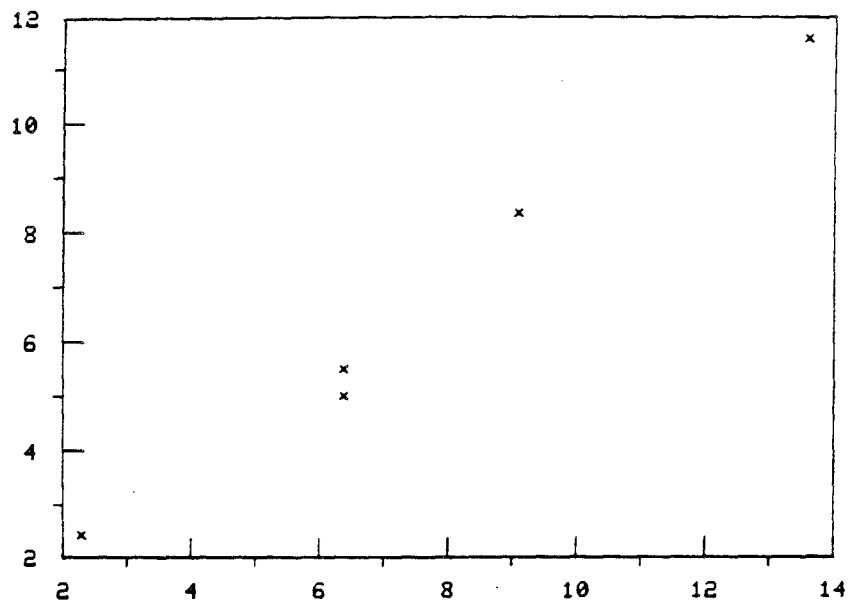


SPLITTING TENSILE STRENGTH OF CONCRETE



EXAMPLE 3

```
COMMENT EXAMPLE--DAVE KLEIN FIRE RESEARCH STUDY
COMMENT MODEL  --LINEAR
COMMENT NOTE   --SQUARE PLOT CHARACTERS AND DOTTED LINES
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.KLEIN1 Y X
.
CHARACTERS X
LINES
PLOT Y X
.
FIT Y = B0+B1*X
.
TITLE FIRE RESEARCH IGNITION STUDY
YLABEL MAX BURN RATE
XLABEL CRIB WEIGHT
X2LABEL MODEL--Y = B0 + B1 * X
X3LABEL JJF6*CS9.NONLINEAR31      JJF6*DATA.KLEIN1      5/78
CHARACTERS SQUARE BLANK
LINES BLANK DASHED
PLOT Y PRED US X
```



LEAST SQUARES NON-LINEAR FIT

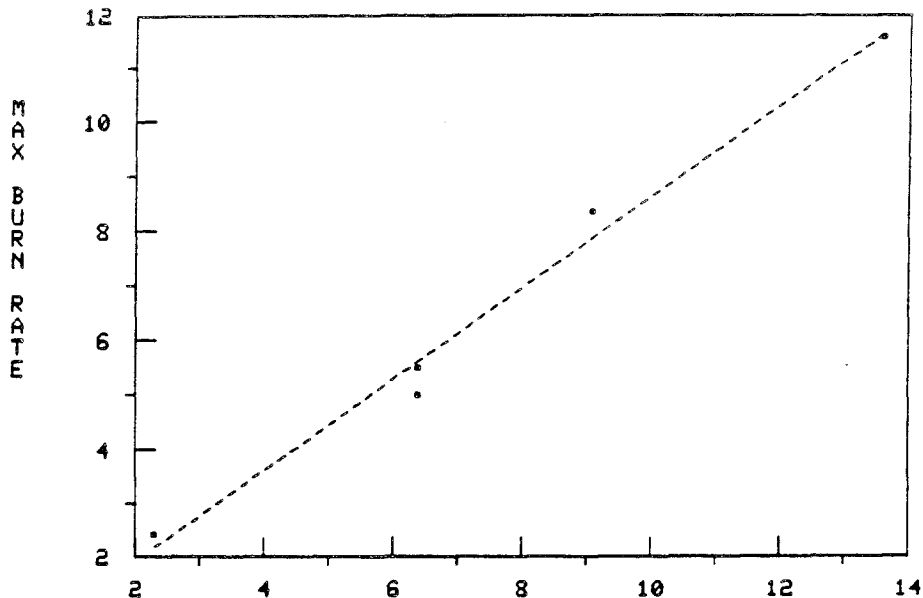
SAMPLE SIZE N = 5
 MODEL-- $Y = B_0 + B_1 * X$
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .3535533920+00
 REPLICATION DEGREES OF FREEDOM = 1
 NUMBER OF DISTINCT SUBSETS = 4

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .27175+01 | .10000+01 .10000+01 |
| 2-- | .50000-02 | .47950+00 | .26015+00 .83597+00 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 B0 .260415 (.4856)
 2 B1 .835924 (.5764-01)

RESIDUAL STANDARD DEVIATION = .4794989079
 RESIDUAL DEGREES OF FREEDOM = 3
 REPLICATION STANDARD DEVIATION = .3535533920
 REPLICATION DEGREES OF FREEDOM = 1
 LACK OF FIT F RATIO = 2.2590 = THE 57.4297% POINT OF THE
 F DISTRIBUTION WITH 2 AND 1 DEGREES OF FREEDOM

FIRE RESEARCH IGNITION STUDY



MODEL-- $Y = B_0 + B_1 * X$
 JJF6*CS9.NONLINEAR31 JJF6*DATA.KLEIN1 5/78

EXAMPLE 4

COMMENT EXAMPLE--BOB MAY OIL CONSUMPTION STUDY
 COMMENT MODEL --LINEAR
 COMMENT NOTE --CIRCLES FOR PLOT CHARACTERS

•
 ECHO ON
 HARDCOPY ON
 BELL ON

•
 READ JJF6*DATA.MAY1 DAY BH1 BH2 OIL

•
 LET H = BH1 + BH2

•
 CHARACTERS CIRCLE
 LINES
 PLOT OIL H

•
 FIT OIL = A + B*H

•
 TITLE OIL TANK LEVEL VS OIL BURNER HOURS

YLABEL TANK LEVEL /GAL/

XLABEL BURNER HOURS

X2LABEL MODEL--Y = A + B*H

X3LABEL JJF6*CS9.NONLINEAR20

JJF6*DATA.MAY1

11/7/78

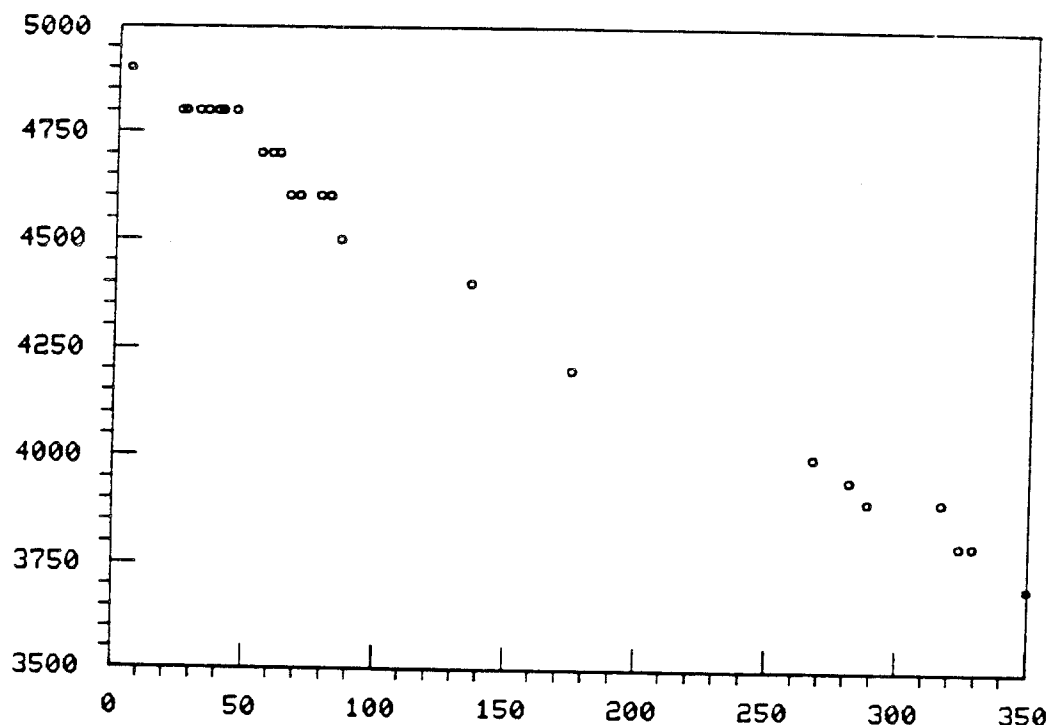
CHARACTER CIRCLE BLANK

LINES BLANK SOLID

PLOT OIL PRED VS H

•
 YLABEL RESIDUALS

PLOT RES H



LEAST SQUARES NON-LINEAR FIT

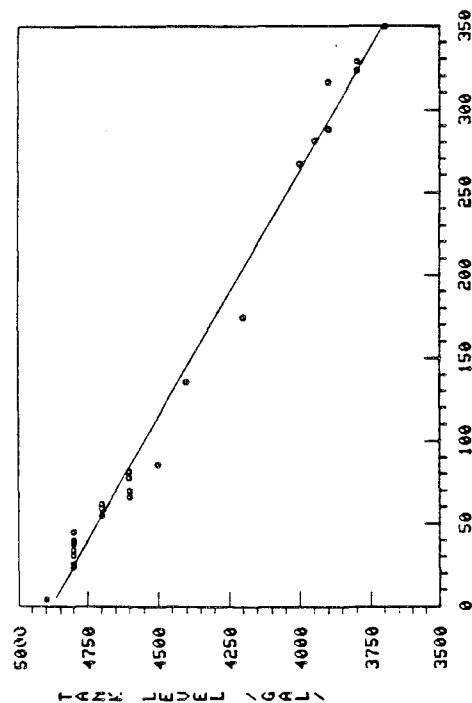
SAMPLE SIZE N = 27
 MODEL-- OIL = A + B*H
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .0000000000
 REPLICATION DEGREES OF FREEDOM = 2
 NUMBER OF DISTINCT SUBSETS = 25

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .44979+04 | .10000+01 |
| 2-- | .50000-02 | .42803+02 | .48854+04 |
| 3-- | .25000-02 | .42791+02 | .48865+04 |

FINAL PARAMETER ESTIMATES
 1 A 4886.54
 2 B -3.36768
 (APPROX. ST. DEV.)
 (12.33)
 (.6793-01)

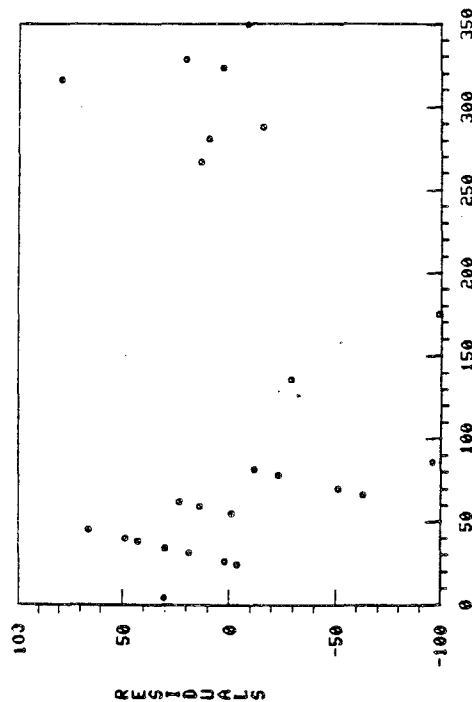
RESIDUAL STANDARD DEVIATION = 42.7909731865
 RESIDUAL DEGREES OF FREEDOM = 25
 REPLICATION STANDARD DEVIATION = .00000000000
 REPLICATION DEGREES OF FREEDOM = 2
 LACK OF FIT F RATIO = ***** - THE 100.0000% POINT OF THE
 F DISTRIBUTION WITH 23 AND 2 DEGREES OF FREEDOM

OIL TANK LEVEL VS OIL BURNER HOURS



BURNER HOURS
 MODEL--Y = A + B*H
 JJF61CS9.NONLINEAR20 JJF61DATA.MAY1 11/7/78

OIL TANK LEVEL VS OIL BURNER HOURS



BURNER HOURS
 MODEL--Y = A + B*H
 JJF61CS9.NONLINEAR20 JJF61DATA.MAY1 11/7/78

EXAMPLE 5

COMMENT EXAMPLE--CLIFF SPIEGELMAN NUCLEAR SAFEGUARDS CALIBRATION PROBLEM
COMMENT MODEL --LINEAR
COMMENT NOTE --VALUE OF RESIDUAL ANALYSIS

•
ECHO ON
HARDCOPY ON
BELL ON

•
READ JJF6XDATA.SPIEGELMAN1 U P

•
CHARACTERS X
LINES
PLOT P U

•
FIT P = $B_0 + B_1 \times U$

•
TITLE NUCLEAR SAFEGUARDS TANK CALIBRATION

YLABEL PRESSURE

XLABEL VOLUME

X2LABEL P = $B_0 + B_1 \times U$

X3LABEL JJF6XCS9.NONLINEAR32 JJF6XDATA.SPIEGELMAN1 10/30/78

CHARACTERS X BLANK

LINES BLANK SOLID

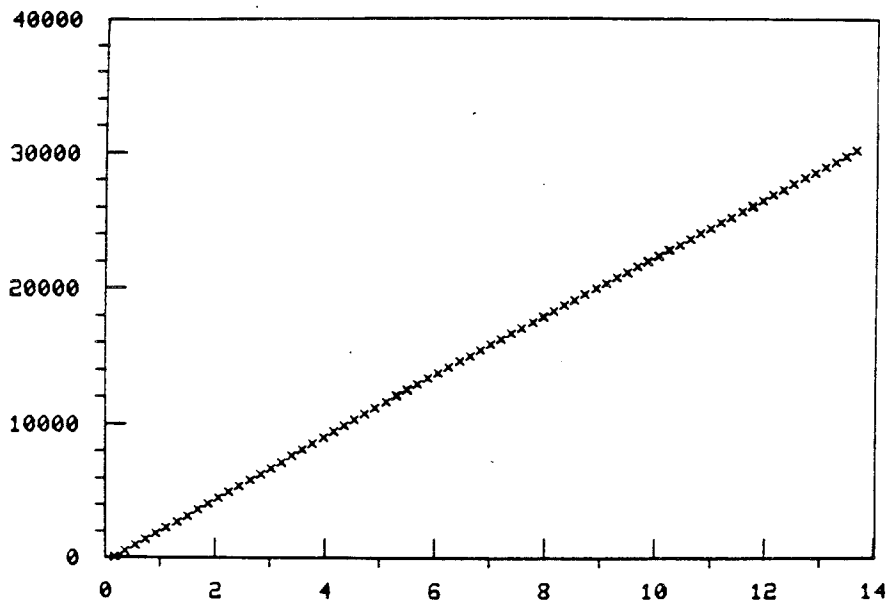
PLOT P PRED VS U

•
YLABEL RESIDUALS

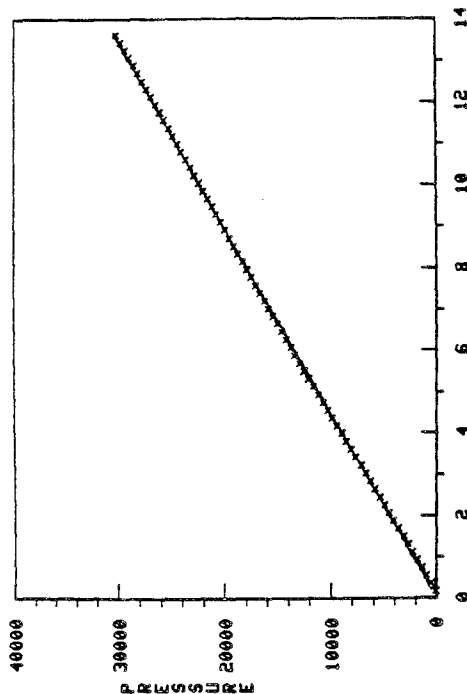
PLOT RES U

•
XLABEL

NORMAL PROBABILITY PLOT RES

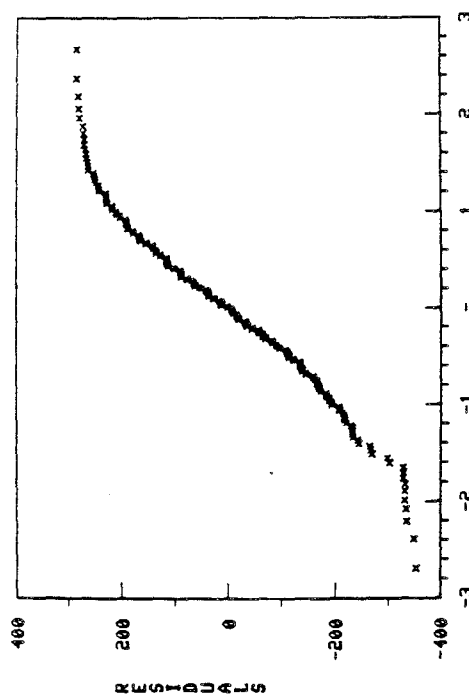


NUCLEAR SAFEGUARDS TANK CALIBRATION



JJF61CS9.NONLINEAR32 JJF61DATA.SPIEGELMAN1 10/30/78
 $P = B_0 + B_1X$

NUCLEAR SAFEGUARDS TANK CALIBRATION



JJF61CS9.NONLINEAR32 JJF61DATA.SPIEGELMAN1 10/30/78
 $P = B_0 + B_1X$

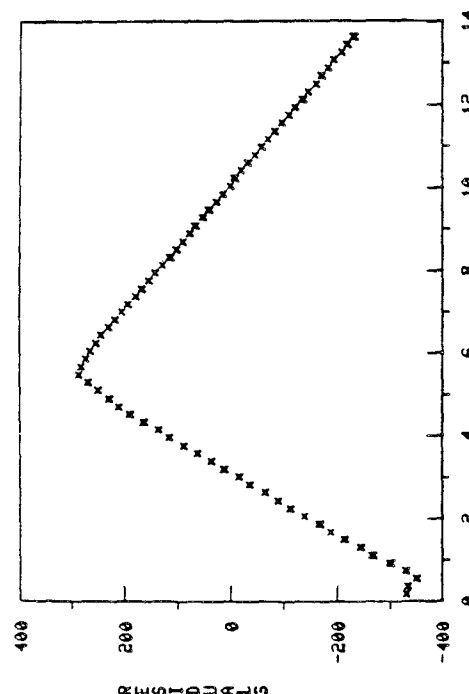
LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 192
 MODEL -- $P = B_0 + B_1X$
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .1874172986+01
 REPLICATION DEGREES OF FREEDOM = 2
 NUMBER OF DISTINCT SUBSETS = 190

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1 | .18000-01 | .18087+05 | .10000+01 |
| 2 | .50000-02 | .17841+03 | .12856+03 |
| 3 | .25000-02 | .17837+03 | .12214+03 |

FINAL PARAMETER ESTIMATES
 1 B0 122.142 (APPROX. ST. DEV.)
 2 B1 2215.65 (26.24)
 (3.249)

RESIDUAL STANDARD DEVIATION = 178.3704223633
 DEGREES OF FREEDOM = 190
 REPLICATION STANDARD DEVIATION = 1.8741729856
 REPLICATION DEGREES OF FREEDOM = 2
 LACK OF FIT F-RATIO = 9154.2249 - THE 99.9891% POINT OF THE F DISTRIBUTION WITH 188 AND 2 DEGREES OF FREEDOM

NUCLEAR SAFEGUARDS TANK CALIBRATION



JJF61CS9.NONLINEAR32 JJF61DATA.SPIEGELMAN1 10/30/78
 $P = B_0 + B_1X$

EXAMPLE 6

COMMENT EXAMPLE--BOB WATTERS CALIBRATION CURVE
COMMENT (SIMULATED DATA)
COMMENT MODEL --QUADRATIC

•
ECHO ON
HARDCOPY ON
BELL ON

•
READ JJF6*DATA.WATTERS1 CONC INT

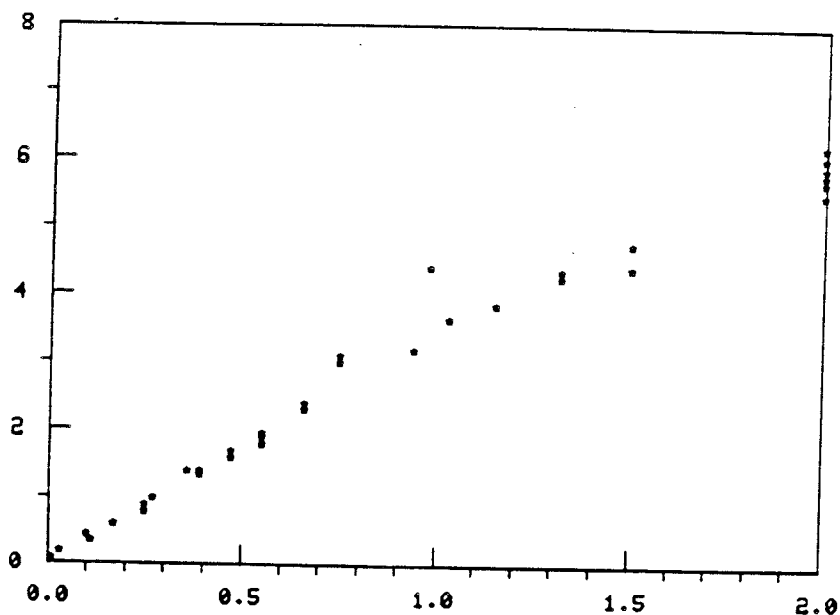
•
CHARACTERS STAR
LINES
PLOT INT CONC

•
FIT INT = $A1*(A0+CONC+A2*CONC**2)$

•
TITLE SI 2881 CALIBRATION CURVE 30JUN78 (SIMULATED DATA)
YLABEL INTENSITY RATIO
XLABEL CONCENTRATION
X2LABEL MODEL--INT = $A1 * (A0 + CONC + A2 * CONC**2)$
X3LABEL JJF6*CS9.NONLINEAR19 JJF6*DATA.WATTERS1 6/78
CHARACTERS STAR BLANK
LINES BLANK SOLID
PLOT INT PRED VS CONC

•
YLABEL RESIDUALS
PLOT RES CONC

•
XLABEL
NORMAL PROBABILITY PLOT RES

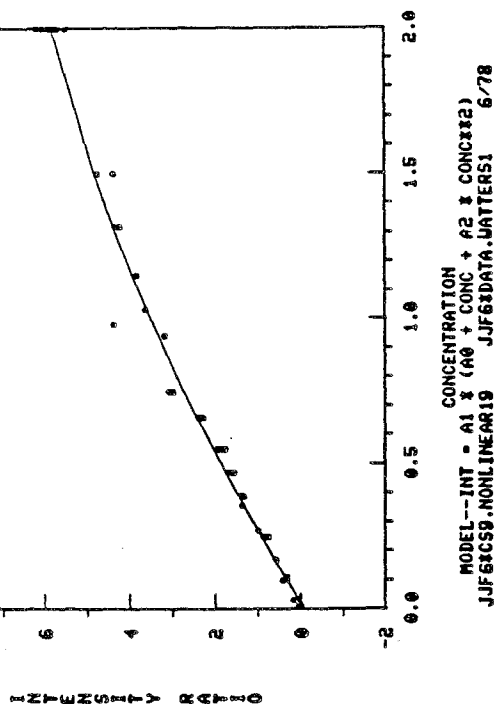


LEAST SQUARES NON-LINEAR FIT

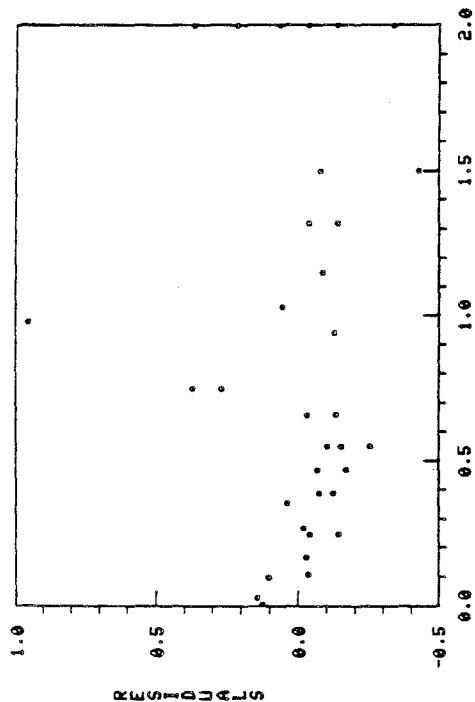
SAMPLE SIZE N = 34
 MODEL--INT = A1*(A0+CONC+A2*CONC**2)
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .1713913567*00
 REPLICATION DEGREES OF FREEDOM = 14
 NUMBER OF DISTINCT SUBSETS = 20

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .71581+00 | .10000+01 |
| 2-- | .11391+00 | .62338+00 | .12246+01 |
| 3-- | .17086+00 | .53122+00 | .15001+01 |
| 4-- | .85438-01 | .48051+00 | .17801+01 |
| 5-- | .19222+00 | .44010+00 | .21029+01 |
| 6-- | .96118-01 | .39478+00 | .24360+01 |
| 7-- | .14416+00 | .35440+00 | .29745+01 |
| 8-- | .72081-01 | .28463+00 | .34915+01 |
| 9-- | .36041-01 | .26024+00 | .40468+01 |
| 10-- | .18020-01 | .24820+00 | .41930+01 |
| 11-- | .90102-02 | .24819+00 | .41991+01 |
| | | | .10000+01 |
| | | | .72164+00 |
| | | | .54107+00 |
| | | | .39529+00 |
| | | | .29492+00 |
| | | | .13369+00 |
| | | | .55575-01 |
| | | | -.44134-01 |
| | | | -.95845-01 |
| | | | -.14390+00 |
| | | | -.14829+00 |
| | | | -.14856+00 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 A1 4.19913
 2 A0 -.167100-01
 3 A2 -.148567
 RESIDUAL STANDARD DEVIATION = .2481906856
 DEGREES OF FREEDOM = 31
 REPLICATION STANDARD DEVIATION = .1713913567
 REPLICATION DEGREES OF FREEDOM = 14
 LACK OF FIT F RATIO = 3.0004 - THE 97.8272% POINT OF THE F DISTRIBUTION WITH 17 AND 14 DEGREES OF FREEDOM

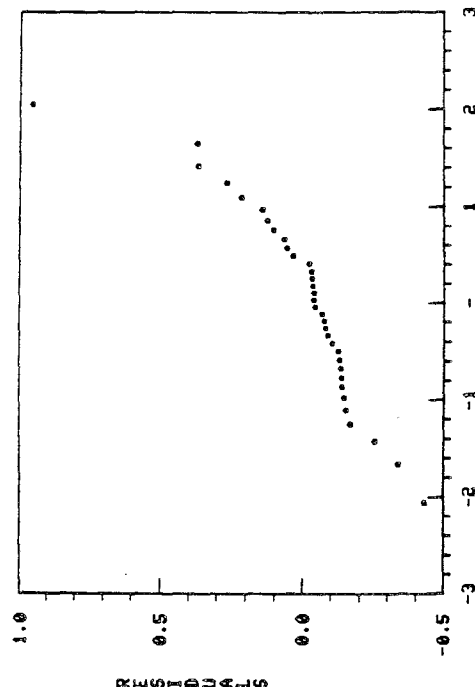


SI 2881 CALIBRATION CURVE 30JUN78 (SIMULATED DATA)



MODEL--INT = A1 * (A0 + CONC + A2 * CONC**2)
 JJF68CS9.NONLINEAR19 JJF68DATA.UATERS1 6/78

SI 2881 CALIBRATION CURVE 30JUN78 (SIMULATED DATA)



MODEL--INT = A1 * (A0 + CONC + A2 * CONC**2)
 JJF68CS9.NONLINEAR19 JJF68DATA.UATERS1 6/78

EXAMPLE 7

COMMENT EXAMPLE--PAUL PONTIUS LOAD CELL CALIBRATION
 COMMENT MODEL --LINEAR AND QUADRATIC
 COMMENT NOTE --VALUE OF RESIDUAL ANALYSIS
 COMMENT NOTE --FITTING, HISTOGRAMS, AND PROB. PLOTS FOR PARTIAL DATA
 COMMENT NOTE --STORING AND PRINTING COEF. AND RESIDUAL STANDARD DEV.

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6XDATA.VARNER2 Y X

CHARACTERS X
 LINES
 PLOT Y X
 PRINT Y X
 LET X=X/10000
 PLOT Y

FIT Y = A0+A1X

TITLE LOAD CELL CALIBRATION
 YLABEL DEFLECTION
 XLABEL LOAD
 X2LABEL MODEL--Y = A0 + A1X
 X3LABEL JJF6XCS9.NONLINEAR12
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Y PRED US X

JJF6XDATA.VARNER2 11/77

YLABEL RESIDUALS
 PLOT RES X
 XLABEL
 NORMAL PROBABILITY PLOT RES

FIT Y = A0+A1X+A2X²

YLABEL DEFLECTION
 XLABEL LOAD
 X2LABEL MODEL--Y = A0 + A1X + A2X²
 PLOT Y PRED US X

YLABEL RESIDUALS
 PLOT RES X
 XLABEL
 NORMAL PROBABILITY PLOT RES
 LET SBOTH=RESSD

FIT Y = B0+B1X+B2X² FOR I = 1 1 20

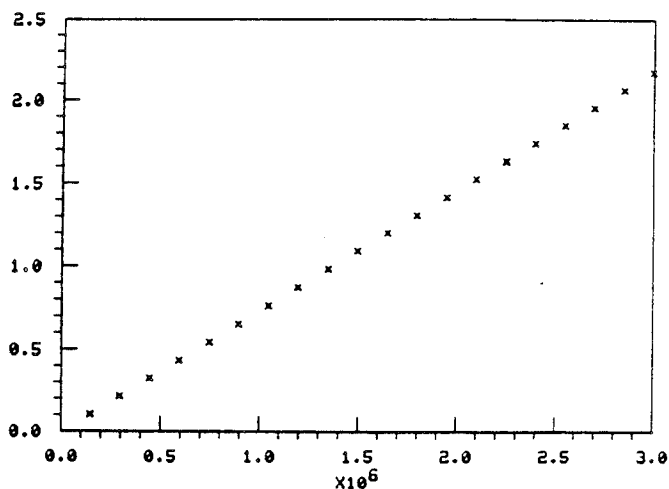
YLABEL DEFLECTION
 XLABEL LOAD
 X2LABEL MODEL Y = B0 + B1X + B2X² (FIRST 20 OBSERVATIONS ONLY)
 PLOT Y PRED US X FOR I = 1 1 20

YLABEL RESIDUALS
 PLOT RES X FOR I = 1 1 20
 XLABEL
 NORMAL PROBABILITY PLOT RES FOR I = 1 1 20
 LET S1=RESSD

FIT Y = C0+C1X+C2X² FOR I = 21 1 40

YLABEL DEFLECTION
 XLABEL LOAD
 X2LABEL MODEL Y = C0 + C1X + C2X² (LAST 20 OBSERVATIONS ONLY)
 PLOT Y PRED US X FOR I = 21 1 40
 YLABEL RESIDUALS
 PLOT RES X FOR I = 21 1 40
 XLABEL
 NORMAL PROBABILITY PLOT RES FOR I = 21 1 40
 LET S2=RESSD

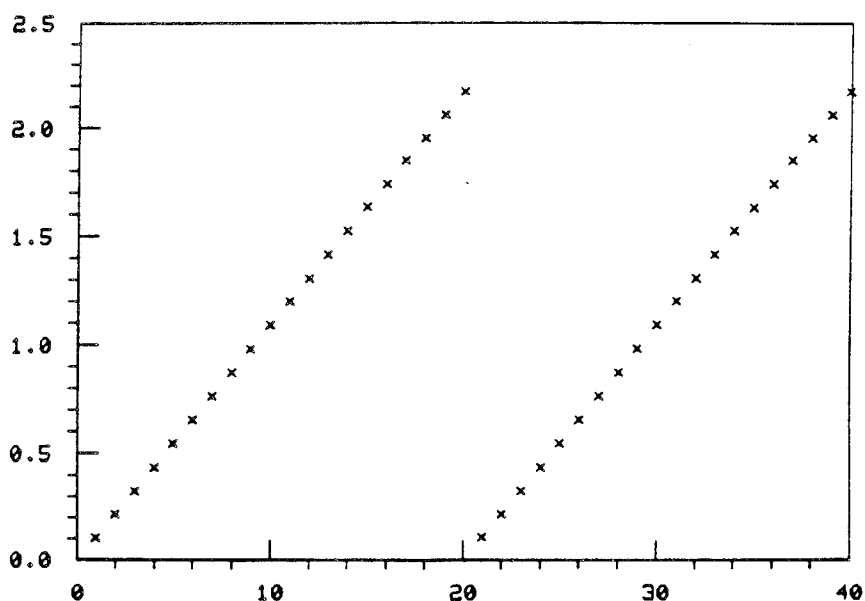
PRINT A0 B0 C0 A1 B1 C1 A2 B2 C2 SBOTH S1 S2

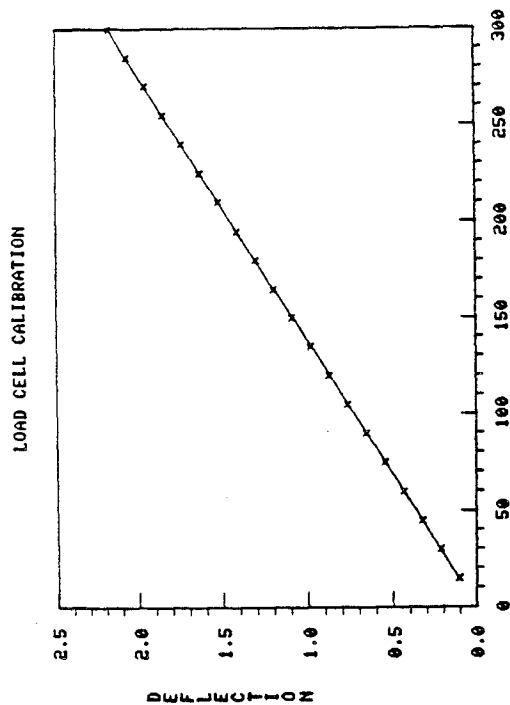


VARIABLES-- Y

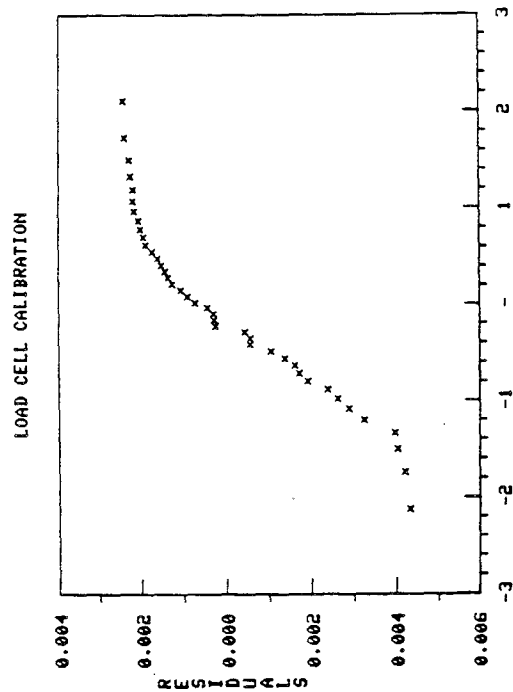
X

| | | |
|------|-------------|-------------|
| 1-- | .1101900+00 | .1500000+06 |
| 2-- | .2195600+00 | .3000000+06 |
| 3-- | .3294900+00 | .4500000+06 |
| 4-- | .4389900+00 | .6000000+06 |
| 5-- | .5480300+00 | .7500000+06 |
| 6-- | .6569400+00 | .9000000+06 |
| 7-- | .7656200+00 | .1050000+07 |
| 8-- | .8748700+00 | .1200000+07 |
| 9-- | .9829200+00 | .1350000+07 |
| 10-- | .1091460+01 | .1500000+07 |
| 11-- | .1200010+01 | .1650000+07 |
| 12-- | .1308220+01 | .1800000+07 |
| 13-- | .1415990+01 | .1950000+07 |
| 14-- | .1523990+01 | .2100000+07 |
| 15-- | .1631940+01 | .2250000+07 |
| 16-- | .1739470+01 | .2400000+07 |
| 17-- | .1846460+01 | .2550000+07 |
| 18-- | .1953920+01 | .2700000+07 |
| 19-- | .2061280+01 | .2850000+07 |
| 20-- | .2168440+01 | .3000000+07 |
| 21-- | .1105200+00 | .1500000+06 |
| 22-- | .2201800+00 | .3000000+06 |
| 23-- | .3293900+00 | .4500000+06 |
| 24-- | .4388600+00 | .6000000+06 |
| 25-- | .5479800+00 | .7500000+06 |
| 26-- | .6573900+00 | .9000000+06 |
| 27-- | .7659600+00 | .1050000+07 |
| 28-- | .8747400+00 | .1200000+07 |
| 29-- | .9830000+00 | .1350000+07 |
| 30-- | .1091500+01 | .1500000+07 |
| 31-- | .1200040+01 | .1650000+07 |
| 32-- | .1308180+01 | .1800000+07 |
| 33-- | .1416130+01 | .1950000+07 |
| 34-- | .1524080+01 | .2100000+07 |
| 35-- | .1631590+01 | .2250000+07 |
| 36-- | .1739650+01 | .2400000+07 |
| 37-- | .1846960+01 | .2550000+07 |
| 38-- | .1954450+01 | .2700000+07 |
| 39-- | .2061770+01 | .2850000+07 |
| 40-- | .2168290+01 | .3000000+07 |





LOAD CELL CALIBRATION
 MODEL--Y = A0 + A1X
 JJF63CS9.NONLINEAR12 JJF63DATA.UARNER2 11/77



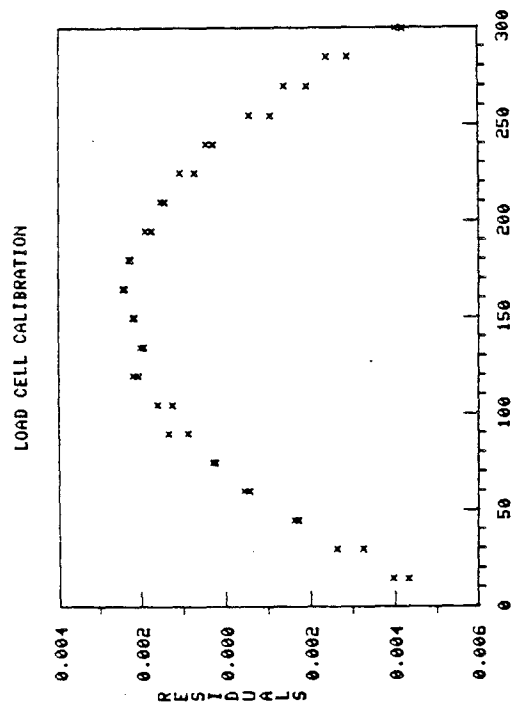
LOAD CELL CALIBRATION
 MODEL--Y = A0 + A1X
 JJF63CS9.NONLINEAR12 JJF63DATA.UARNER2 11/77

LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 40
 MODEL Y = A0 + A1X
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .2147274445-03
 REPLICATION DEGREES OF FREEDOM = 20
 NUMBER OF DISTINCT SUBSETS = 20

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .18392+03 | .10000+01 |
| 2-- | .50000-02 | .37569-01 | -.60833-01 |
| 3-- | .25000-02 | .21713-02 | .61355-02 |
| 4-- | .12500-02 | .21713-02 | .61498-02 |

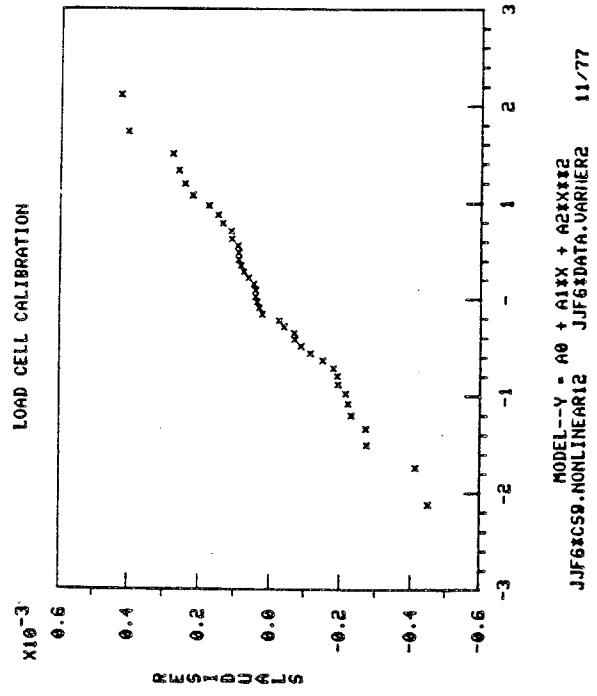
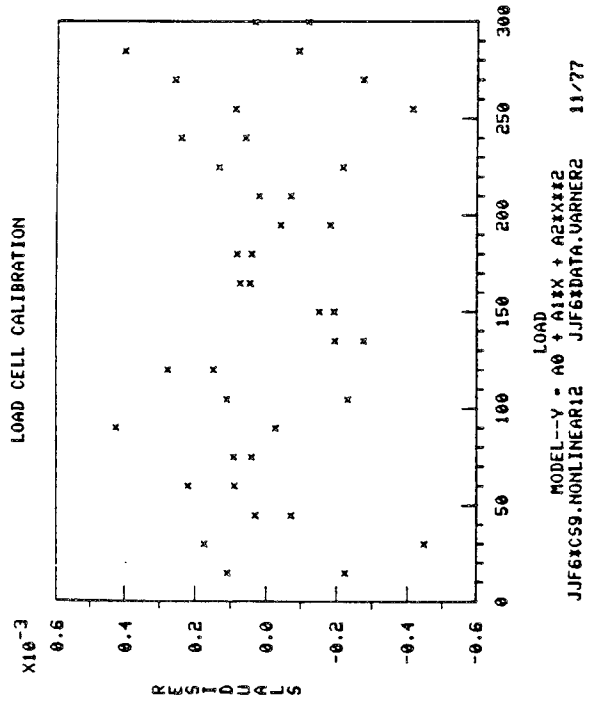
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 A0 .614928-02
 2 A1 .722103-02 (.7132-03) (.3969-05)

RESIDUAL STANDARD DEVIATION = .0021712666
 RESIDUAL DEGREES OF FREEDOM = 38
 REPLICATION STANDARD DEVIATION = .0002147274
 REPLICATION DEGREES OF FREEDOM = 20
 LACK OF FIT F RATIO = 214.7440 = THE 100-0000% POINT OF THE F DISTRIBUTION WITH 18 AND 20 DEGREES OF FREEDOM



LOAD CELL CALIBRATION
 MODEL--Y = A0 + A1X
 JJF63CS9.NONLINEAR12 JJF63DATA.UARNER2 11/77

| LEAST SQUARES NON-LINEAR FIT | | | | | | | | | |
|-------------------------------------|---------------------|-----------------------------|----------------------|---------------------|---------------------------|------------|--|--|--|
| SAMPLE SIZE N = 40 | | | | | | | | | |
| MODEL --- Y = A0 + A1X + A2X**2 | | | | | | | | | |
| REPLICATION CASE | | | | | | | | | |
| REPLICATION STANDARD DEVIATION = | | | | | | | | | |
| REPLICATION DEGREES OF FREEDOM = 20 | | | | | | | | | |
| NUMBER OF DISTINCT SUBSETS | | | | | | | | | |
| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | % STANDARD DEVIATION | PARAMETER ESTIMATES | | | | | |
| 1-- | .10000-01 | .44740+05 | × | .61493-02 | .72310-02 | .10000+01 | | | |
| 2-- | .50000-02 | .2418+02 | × | .14046+02 | -.60841+00 | .25395-02 | | | |
| 3-- | .25000-02 | .43889-01 | × | .12114+00 | .52749-02 | .59413-05 | | | |
| 4-- | .12500-02 | .20802-03 | × | .76938-03 | .73190-02 | -.31122-06 | | | |
| 5-- | .62500-03 | .20518-03 | × | .67368-03 | .73206-02 | -.31607-06 | | | |
| FINAL PARAMETER ESTIMATES | | | | | (APPROX. ST. DEV.) | | | | |
| 1 A0 | .673531-03 | | | | (.1079-03) | | | | |
| 2 A1 | .732059-02 | | | | (.1578-05) | | | | |
| 3 A2 | -.316083-06 | | | | (.4067-08) | | | | |
| RESIDUAL STANDARD DEVIATION = | | | | | .0002051765 | | | | |
| DEGREES OF FREEDOM = | | | | | 37 | | | | |
| REPLICATION STANDARD DEVIATION = | | | | | .0002147274 | | | | |
| REPLICATION DEGREES OF FREEDOM = | | | | | 20 | | | | |
| LACK OF FIT F RATIO = | | | | | .8107 | | | | |
| F DISTRIBUTION WITH | | | | | 17 AND | | | | |
| | | | | | THE 33.3796% POINT OF THE | | | | |
| | | | | | 20 DEGREES OF FREEDOM | | | | |

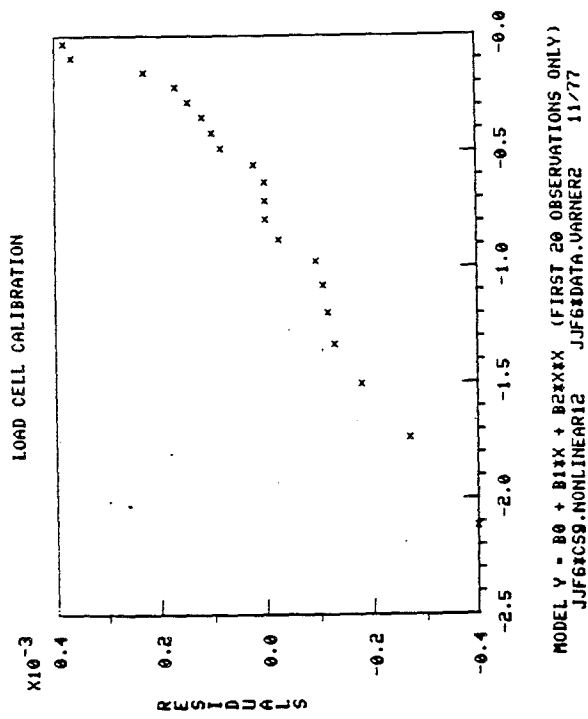
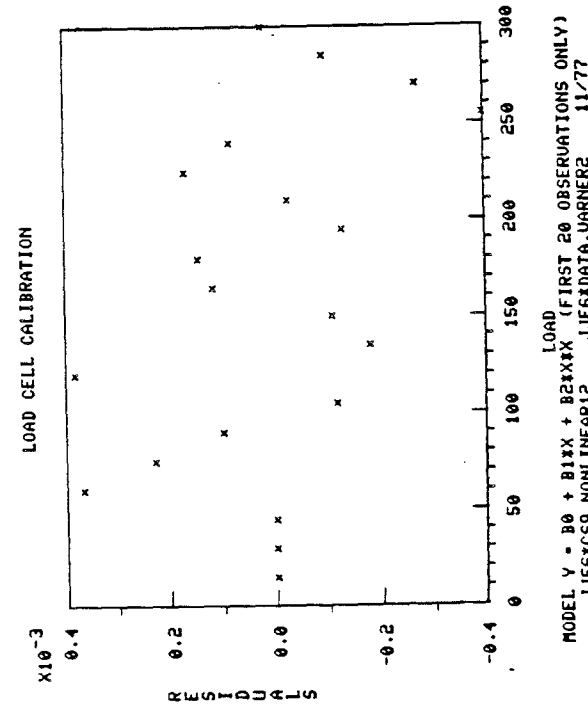
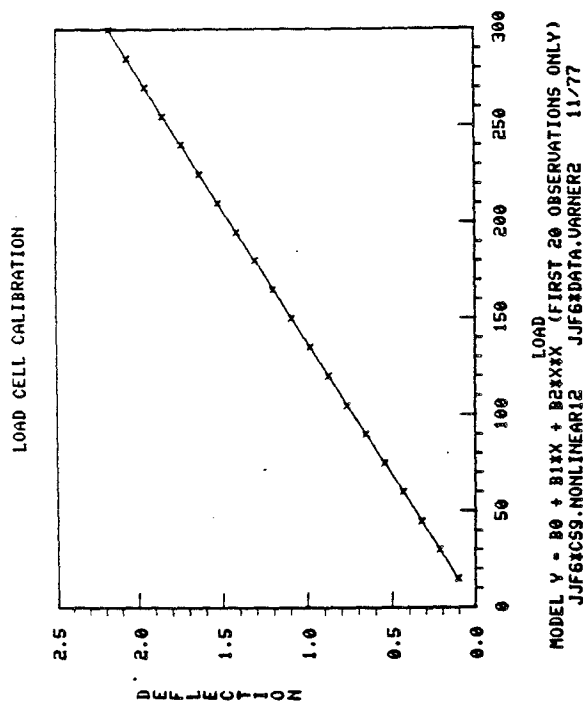


LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 20
 MODEL -- Y = B0 + B1X + B2X^2
 NO REPLICATION CASE

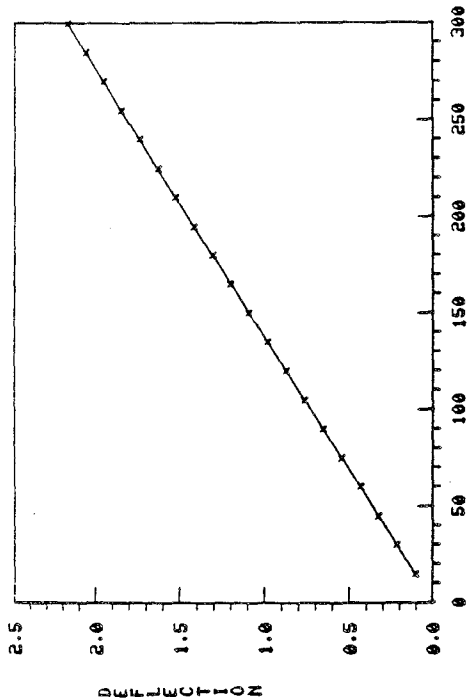
| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------------------|
| 1 | .10000-01 | .46528+05 | .10000+01 .10000+01 .10000+01 |
| 2 | .10000-03 | .38731+02 | -.58315+02 -.12348+01 .39696-02 |
| 3 | .25000-03 | .66342-01 | .83359+00 .34298-02 .11494-04 |
| 4 | .12500-03 | .21712-03 | .67197-03 .73196-02 -.31354-06 |
| 5 | .62500-03 | .20640-03 | .49040-03 .73227-02 -.32271-06 |
| 6 | .12014-01 | .20640-03 | .49100-03 .73226-02 -.32268-06 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 B0 .490636-03 (.1537-03)
 2 B1 .732265-02 (.2247-05)
 3 B2 -.322695-06 (.6926-08)

RESIDUAL STANDARD DEVIATION = .0002064030
 RESIDUAL DEGREES OF FREEDOM = 17

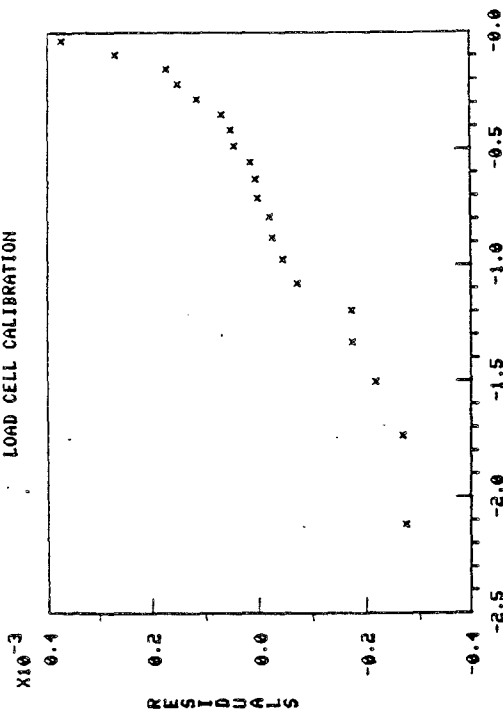


LOAD CELL CALIBRATION



MODEL $Y = C0 + C1X + C2X^2$ (LAST 20 OBSERVATIONS ONLY)
JJF6XCS9.NONLINEAR12 JJF6XDATA.VARNER2 11/77

LOAD CELL CALIBRATION



MODEL $Y = C0 + C1X + C2X^2$ (LAST 20 OBSERVATIONS ONLY)
JJF6XCS9.NONLINEAR12 JJF6XDATA.VARNER2 11/77

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 20
MODEL-- $Y = C0 + C1X + C2X^2$
NO REPLICATION CASE

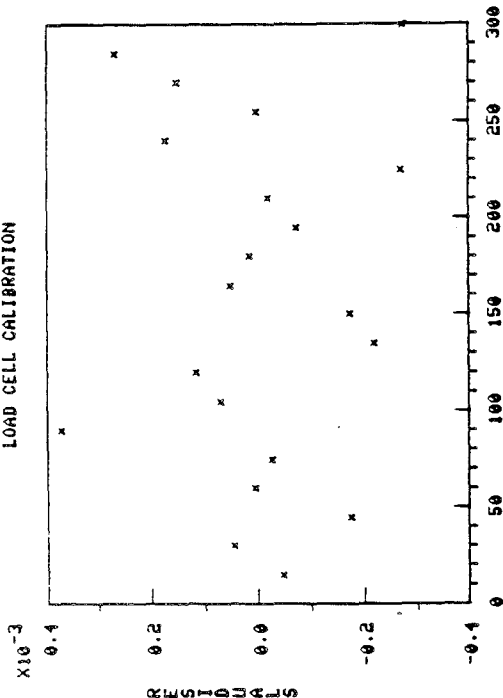
| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1 | .10000-01 | .46578+05 | .10000+01 |
| 2 | .50000-02 | .28731+02 | .68315+02 |
| 3 | .25000-02 | .86963-01 | .23412+00 |
| 4 | .12500-02 | .19213-03 | .10386-02 |
| 5 | .62500-03 | .17981-03 | .85651-03 |
| 6 | .35596-02 | .17980-03 | .85639-03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

| | | | |
|---|----|-------------|------------|
| 1 | C0 | .856317-03 | (.1337-03) |
| 2 | C1 | .731853-02 | (.1955-05) |
| 3 | C2 | -.309474-06 | (.6025-08) |

RESIDUAL STANDARD DEVIATION = .8001798039
DEGREES OF FREEDOM = 17

LOAD CELL CALIBRATION



MODEL $Y = C0 + C1X + C2X^2$ (LAST 20 OBSERVATIONS ONLY)
JJF6XCS9.NONLINEAR12 JJF6XDATA.VARNER2 11/77

PARAMETERS AND CONSTANTS--

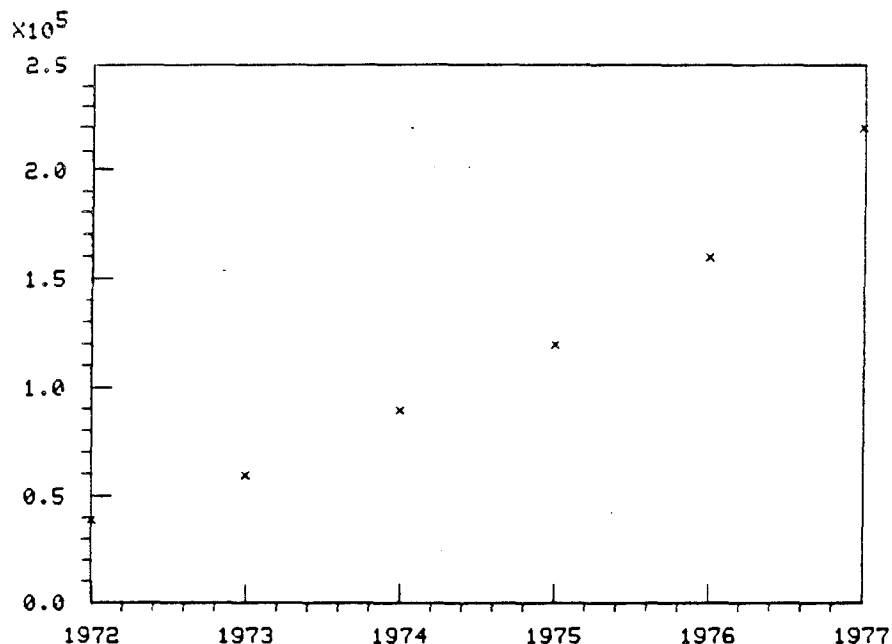
| | |
|----|--------------|
| A0 | -.6735314-03 |
| B0 | -.4910012-03 |
| C0 | -.8563168-03 |
| A1 | .7320692-02 |
| B1 | .7322649-02 |
| C1 | .7318532-02 |
| A2 | -.0828-06 |
| B2 | .827-06 |
| C2 | .756-06 |

EXAMPLE 8

```

COMMENT EXAMPLE--JEAN YANCEY COMPUTER UTILIZATION STUDY
COMMENT MODEL  --AN EXPONENTIAL
COMMENT NOTE   --EXTRAPOLATION
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF6*DATA.YANCEY3 YEAR NUMCPU
.
CHARACTERS X
LINES
PLOT NUMCPU YEAR
.
LET MY=NUMCPU(6)
LET MX=YEAR(6)
LET A0=0
LET A1=1
LET A2=ALOG(MY)/(MX-1950)
FIT NUMCPU = A0+A1*EXP(A2*(YEAR-1950))
.
TITLE MINI-COMPUTER UTILIZATION STUDY (UNITED STATES)
YLABEL NUMBER OF CPU'S
XLABEL YEAR
X2LABEL MODEL--Y = A0 + A1 * EXP(A2 * (YEAR-1950))
X3LABEL JJF6*CS9.NONLINEAR24 JJF6*DATA.YANCEY3 11/1/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT NUMCPU PRED US YEAR
.
PLOT NUMCPU YEAR AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1972 1 1985
.
CHARACTERS X BLANK BLANK
LINES BLANK SOLID DOTTED
PLOT NUMCPU YEAR AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1972 1 1977 AND
PLOT Y = A0+A1*EXP(A2*(X-1950)) FOR X = 1977 1 1985

```



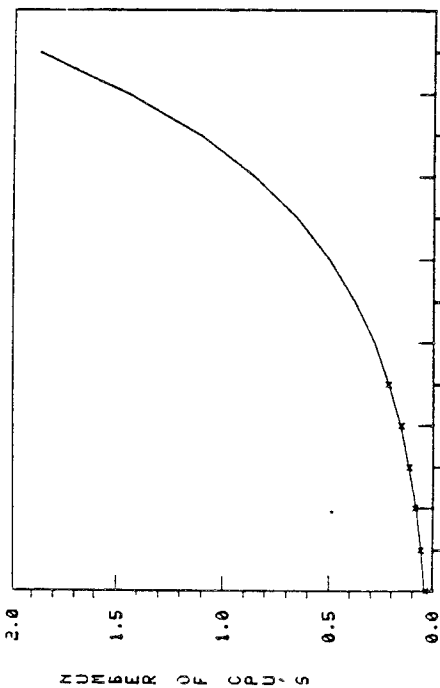
LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 6
 MODEL--NUNCPU = A0+A1*EXP(A2*(YEAR-1950))
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|--------------------|---------------------|
| 1-- | .10000-01 | .24002+05 | .00000 |
| 2-- | .25500-01 | .23972+05 | .20956+05 |
| 3-- | .33750-01 | .11854+05 | .24147+05 |
| 4-- | .16875-01 | .10152+05 | .21888+05 |
| 5-- | .37969-01 | .86752+04 | .20203+05 |
| 6-- | .18781-01 | .81540+04 | .18542+05 |
| 7-- | .21317-01 | .71800+04 | .17035+05 |
| 8-- | .48051-01 | .72635+04 | .16255+05 |
| 9-- | .24027-01 | .68083+04 | .14164+05 |
| 10-- | .36041-01 | .65141+04 | .12521+05 |
| 11-- | .18020-01 | .62214+04 | .11005+05 |
| 12-- | .46546-01 | .59267+04 | .87057+04 |
| 13-- | .20273-01 | .59105+04 | .67471+04 |
| 14-- | .74202-01 | .52311+04 | .39593+04 |
| 15-- | .31210-01 | .52311+04 | .23177+04 |
| 16-- | .11105-01 | .50355+04 | .23177+04 |
| 17-- | .26688-01 | .50173+04 | .12488+04 |
| 18-- | .26688-01 | .46832+04 | .12488+04 |
| 19-- | .28855-01 | .43251+04 | .32673+04 |
| 20-- | .17431-01 | .41747+04 | .46620+04 |
| 21-- | .17431-01 | .39138+04 | .59571+04 |
| 22-- | .17431-01 | .39138+04 | .72023+04 |
| 23-- | .23355-01 | .37810+04 | .83732+04 |
| 24-- | .13177-01 | .36672+04 | .98572+04 |
| 25-- | .27309-01 | .35091+04 | .10947+05 |
| 26-- | .17000-01 | .34230+04 | .11900+05 |
| 27-- | .28549-01 | .33341+04 | .13663+05 |
| 28-- | .10275-01 | .32196+04 | .15118+05 |
| 29-- | .27168-01 | .30172+04 | .15153+05 |
| 30-- | .17330-01 | .29555+04 | .16133+05 |
| 31-- | .56033-02 | .29555+04 | .16133+05 |
| 32-- | .13044-01 | .29363+04 | .18009+05 |
| 33-- | .65030-02 | .28730+04 | .21655+05 |
| 34-- | .97530-02 | .28529+04 | .23988+05 |
| 35-- | .48705-02 | .28243+04 | .24599+05 |
| 36-- | .23362-02 | .28184+04 | .25655+05 |
| 37-- | .18251-02 | .28138+04 | .27103+05 |
| 38-- | .18251-02 | .28138+04 | .27783+05 |
| 39-- | .18251-02 | .28138+04 | .27783+05 |
| 40-- | .18251-02 | .28138+04 | .27783+05 |
| 41-- | .18251-02 | .28138+04 | .27783+05 |
| 42-- | .18251-02 | .28138+04 | .27783+05 |

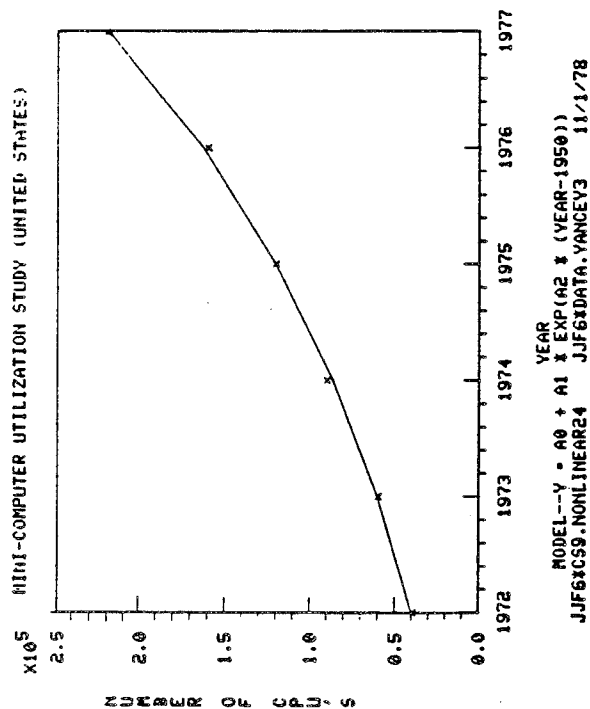
FINAL PARAMETER ESTIMATES
 1 A0 -27910.2 (APPROX. ST. DEV.)
 2 A1 248.650 (190.3)
 3 A2 .255533 (.2548-01)

RESIDUAL STANDARD DEVIATION = 2012.2099267578
 DEGREES OF FREEDOM = 3

X106 MINI-COMPUTER UTILIZATION STUDY (UNITED STATES)

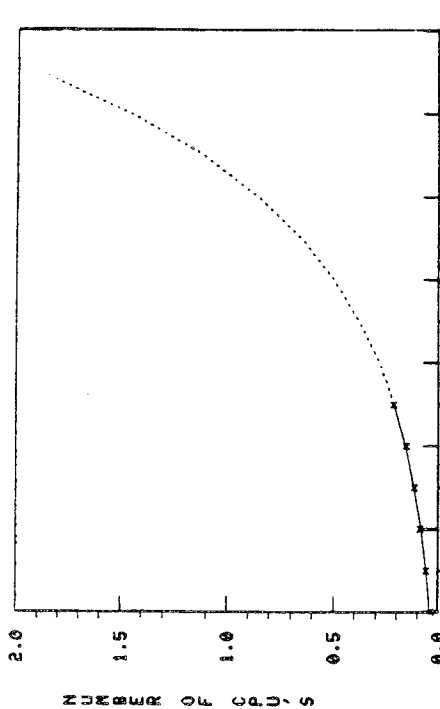


MODEL--Y = A0 + A1 * EXP(A2 * (YEAR-1950))
 JJJF6*CS9.NONLINEAR24 JJJF6*DATA.YANCEY3 11/1/78



MODEL--Y = A0 + A1 * EXP(A2 * (YEAR-1950))
 JJJF6*CS9.NONLINEAR24 JJJF6*DATA.YANCEY3 11/1/78

X106 MINI-COMPUTER UTILIZATION STUDY (UNITED STATES)



MODEL--Y = A0 + A1 * EXP(A2 * (YEAR-1950))
 JJJF6*CS9.NONLINEAR24 JJJF6*DATA.YANCEY3 11/1/78

EXAMPLE 9

COMMENT EXAMPLE--BILL BEINE SAFETY EQUIPMENT STUDY
COMMENT MODEL --AN EXPONENTIAL
COMMENT NOTE --FIT SUBSETS AND SUPERIMPOSE FITTED CURVES

ECHO ON
HARDCOPY ON
BELL ON
VERSATEC ON

READ JJF6*DATA.BEINE6 Y X ID
PRINT 1 2 3

CHARACTERS CIRCLE X STAR TRIANGLE
LINES BLANK ALL
PLOT Y X ID

LET A0=30
LET A1=50
LET A2=1
FIT Y=A0+A1*EXP(-A2*X) SUBSET ID 3

LET B0=50
LET B1=40
LET B2=1
FIT Y=B0+B1*EXP(-B2*X) SUBSET ID 4

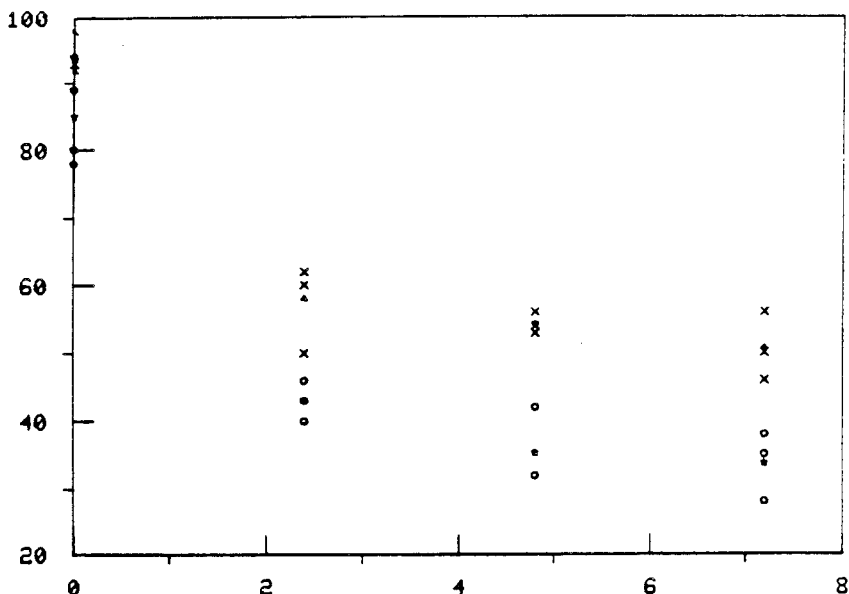
TITLE SAFETY EQUIPMENT STUDY
YLABEL PEAK ACCELERATION
XLABEL MOISTURE CONTENT (KG)
X2LABEL MODEL--Y = C0 + C1 * EXP(-C2 * X)
X3LABEL JJF6*CS9.NONLINEAR14 JJF6*DATA.BEINE6 11/7/78
LINES BLANK BLANK DOTTED DASHED
CHARACTERS CIRCLE X BLANK BLANK

PLOT Y X SUBSET ID 1 AND
PLOT Y X SUBSET ID 2 AND
PLOT Y = A0+A1*EXP(-A2*X) FOR X = 0 .1 8 AND
PLOT Y = B0+B1*EXP(-B2*X) FOR X = 0 .1 8

XLIMITS -2 10

PLOT Y X SUBSET ID 1 AND
PLOT Y X SUBSET ID 2 AND
PLOT Y = A0+A1*EXP(-A2*X) FOR X = 0 .1 8 AND
PLOT Y = B0+B1*EXP(-B2*X) FOR X = 0 .1 8

| VARIABLES-- | 1 | 2 | 3 |
|-------------|------------|------------|------------|
| 1-- | .780000+02 | .000000 | .100000+01 |
| 2-- | .800000+02 | .000000 | .100000+01 |
| 3-- | .940000+02 | .000000 | .100000+01 |
| 4-- | .850000+02 | .000000 | .100000+01 |
| 5-- | .853000+02 | .000000 | .300000+01 |
| 6-- | .400000+02 | .240000+01 | .100000+01 |
| 7-- | .460000+02 | .240000+01 | .100000+01 |
| 8-- | .430000+02 | .240000+01 | .100000+01 |
| 9-- | .430000+02 | .240000+01 | .300000+01 |
| 10-- | .320000+02 | .480000+01 | .100000+01 |
| 11-- | .420000+02 | .480000+01 | .100000+01 |
| 12-- | .320000+02 | .480000+01 | .100000+01 |
| 13-- | .353000+02 | .480000+01 | .300000+01 |
| 14-- | .380000+02 | .720000+01 | .100000+01 |
| 15-- | .280000+02 | .720000+01 | .100000+01 |
| 16-- | .350000+02 | .720000+01 | .100000+01 |
| 17-- | .337000+02 | .720000+01 | .300000+01 |
| 18-- | .980000+02 | .000000 | .200000+01 |
| 19-- | .920000+02 | .000000 | .200000+01 |
| 20-- | .930000+02 | .000000 | .200000+01 |
| 21-- | .943000+02 | .000000 | .400000+01 |
| 22-- | .500000+02 | .240000+01 | .200000+01 |
| 23-- | .600000+02 | .240000+01 | .200000+01 |
| 24-- | .620000+02 | .240000+01 | .200000+01 |
| 25-- | .580000+02 | .240000+01 | .400000+01 |
| 26-- | .530000+02 | .480000+01 | .200000+01 |
| 27-- | .560000+02 | .480000+01 | .200000+01 |
| 28-- | .540000+02 | .480000+01 | .200000+01 |
| 29-- | .543000+02 | .480000+01 | .400000+01 |
| 30-- | .560000+02 | .720000+01 | .200000+01 |
| 31-- | .460000+02 | .720000+01 | .200000+01 |
| 32-- | .500000+02 | .720000+01 | .200000+01 |
| 33-- | .507000+02 | .720000+01 | .400000+01 |

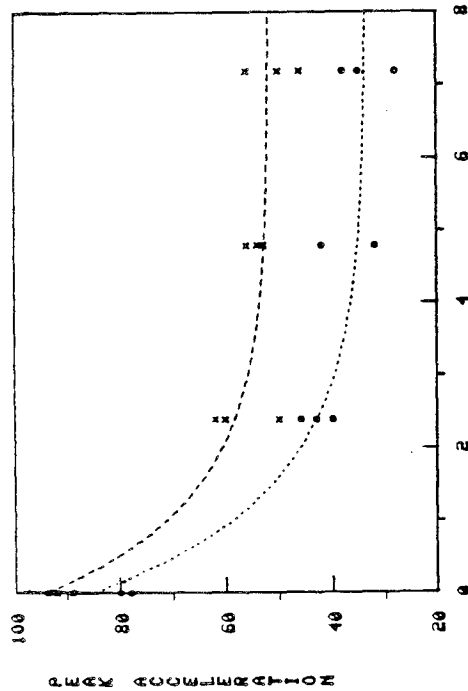


LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 4
 MODEL-- Y=A0+A1*EXP(-A2*X)
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|--|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .11707+02 | .30000+02 |
| 2-- | .50000-02 | .42795+01 | .51517+02 |
| 3-- | .25000-02 | .64436+00 | .51626+02 |
| 4-- | .12500-02 | .11395+00 | .51850+02 |
| 5-- | .62500-03 | .11394+00 | .51850+02 |
| FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.) | | | |
| 1 A0 | 53.4377 | (.1051) | |
| 2 A1 | 51.8691 | (.1529) | |
| 3 A2 | .703326 | (.6990-02) | |
| RESIDUAL STANDARD DEVIATION | .1139446376 | | |
| DEGREES OF FREEDOM | 1 | | |

SAFETY EQUIPMENT STUDY



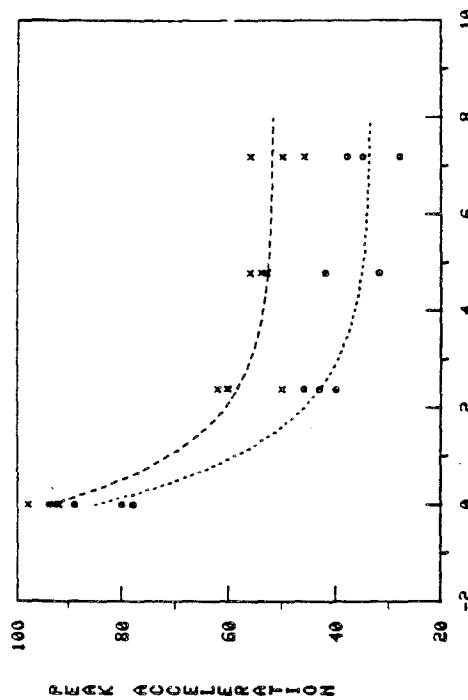
MOISTURE CONTENT (KG)
 MODEL--Y = C0 + C1 * EXP(-C2 * X)
 JJF68C59.NONLINEAR14 JJF68DATA.BEIN6 11/7/78

LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 4
 MODEL-- Y=B0+B1*EXP(-B2*X)
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|--|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .73358+01 | .50000+02 |
| 2-- | .50000-02 | .23856+01 | .51940+02 |
| 3-- | .25000-02 | .19702+01 | .51586+02 |
| 4-- | .12500-02 | .19771+01 | .51705+02 |
| FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.) | | | |
| 1 B0 | 51.7084 | (1.731) | |
| 2 B1 | 42.5631 | (2.802) | |
| 3 B2 | .771186 | (.1700) | |
| RESIDUAL STANDARD DEVIATION | 1.9770912677 | | |
| DEGREES OF FREEDOM | 1 | | |

SAFETY EQUIPMENT STUDY



MOISTURE CONTENT (KG)
 MODEL--Y = C0 + C1 * EXP(-C2 * X)
 JJF68C59.NONLINEAR14 JJF68DATA.BEIN6 11/7/78

EXAMPLE 10

COMMENT EXAMPLE--DOUG SHIER ERYTHEMA STUDY
COMMENT MODEL --SUM OF 2 EXPONENTIALS

•
ECHO ON
HARDCOPY ON
BELL ON

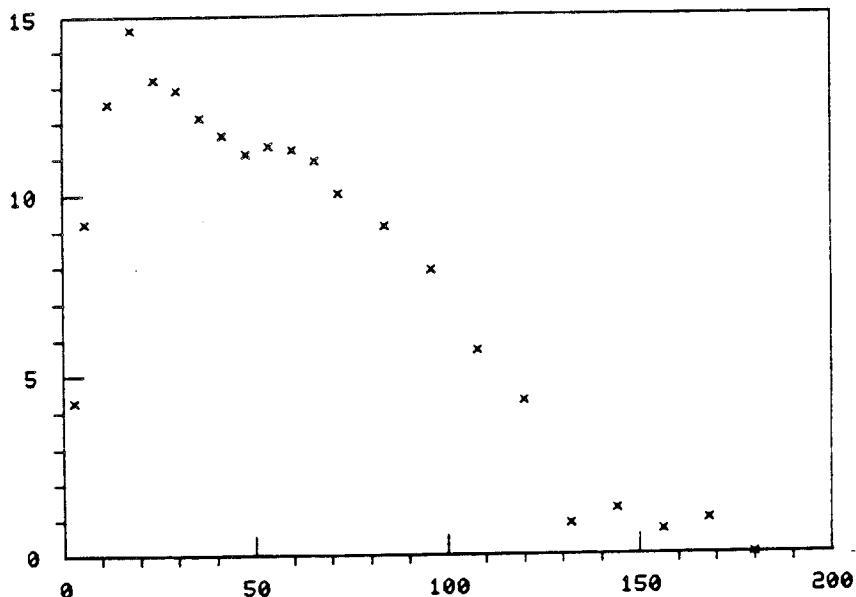
•
SKIP 25
READ JJF6*DATA.SHIER1 REDNESS TIME

•
CHARACTERS X
LINES BLANK
PLOT REDNESS TIME

•
LET A=20
LET B=.1
FIT REDNESS = A * (EXP(-B*TIME/10) - EXP(-TIME/10))

•
TITLE ERYTHEMA (REDNESS OF A SKIN SORE) BIOMEDICAL STUDY
YLABEL REDNESS
XLABEL TIME (IN HOURS)
X2LABEL MODEL--Y = A * ((EXP(-B*X/10) - EXP(-X/10))
X3LABEL JJF6*CS9.NONLINEAR3 JJF6*DATA.SHIER1 11/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT REDNESS PRED VS TIME

•
YLABEL RESIDUALS
PLOT RES TIME



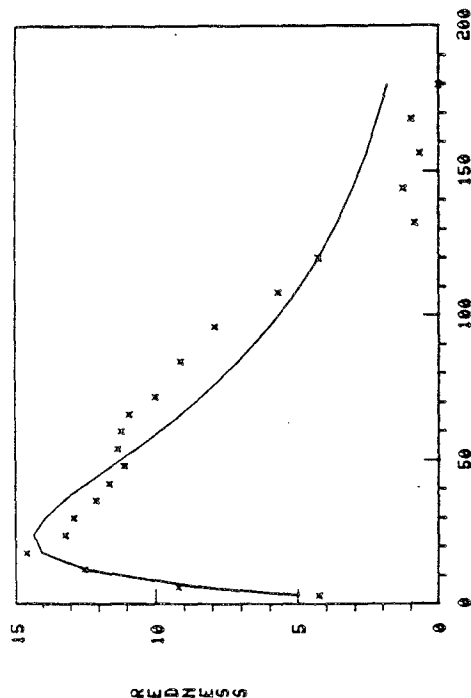
LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 22
 MODEL-- REDNESS = A * (EXP(-B*TIME/10)) - EXP(-TIME/10))
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .19622+01 | .20000+02 |
| 2-- | .50000-02 | .14315+01 | .22889+02 |
| 3-- | .25000-02 | .14312+01 | .23006+02 |
| | | | .10000+00 |
| | | | .13957+00 |
| | | | .14016+00 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 A 23.0060 (1.489)
 2 B .140162 (.1265-01)

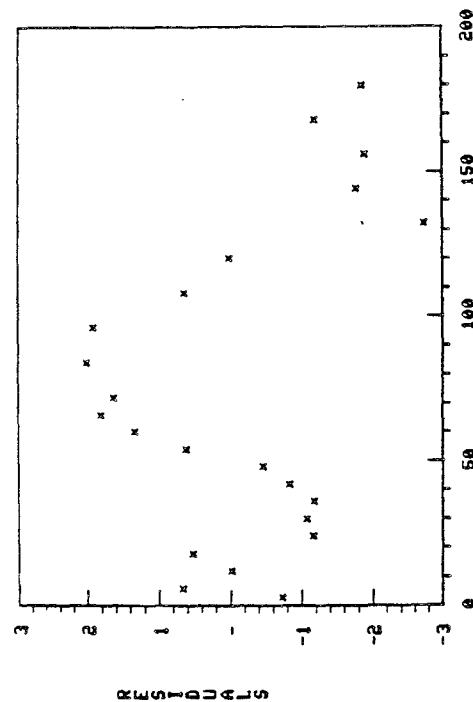
RESIDUAL STANDARD DEVIATION = 1.4311647564
 DEGREES OF FREEDOM = 20

ERYTHEMA (REDNESS OF A SKIN SORE) BIOMEDICAL STUDY



MODEL--Y = A * ((EXP(-B*X/10)) - EXP(-X/10))
 JJF6XCS9.NONLINEAR3 JJF6XDATA.SHIER1 11/76

ERYTHEMA (REDNESS OF A SKIN SORE) BIOMEDICAL STUDY



MODEL--Y = A * ((EXP(-B*X/10)) - EXP(-X/10))
 JJF6XCS9.NONLINEAR3 JJF6XDATA.SHIER1 11/76

EXAMPLE 11

COMMENT EXAMPLE--DOUG SHIER QUEUEING THEORY SERVER FUNCTION PHI(X)
 COMMENT MODEL --POWER FUNCTION
 COMMENT NOTE --FULLY-SPECIFIED MODEL FIT CAPABILITY

ECHO ON
 HARDCOPY ON
 BELL ON

SKIP 25
 READ JJF6*DATA.SHIER2 X Y

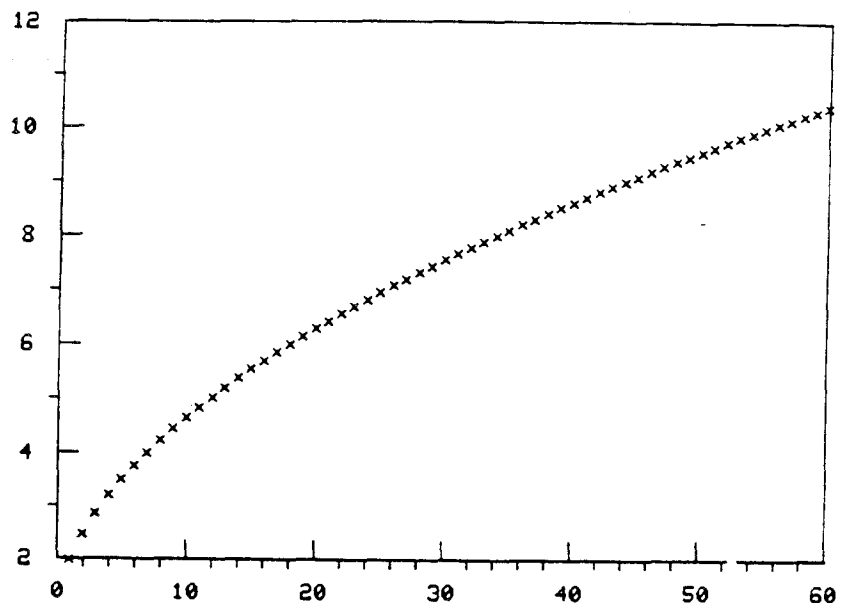
CHARACTERS X
 LINES
 PLOT Y X

CHARACTERS X BLANK
 LINES BLANK SOLID
 FIT Y = $X^{.5}$
 PLOT Y PRED US X
 FIT Y = $1 + X^{.5}$
 PLOT Y PRED US X
 FIT Y = $1 + X^{.7}$
 PLOT Y PRED US X
 FIT Y = $1 + X^{.55}$
 PLOT Y PRED US X

LET B2=.55
 FIT Y = $B_0 + B_1 X^{B2}$

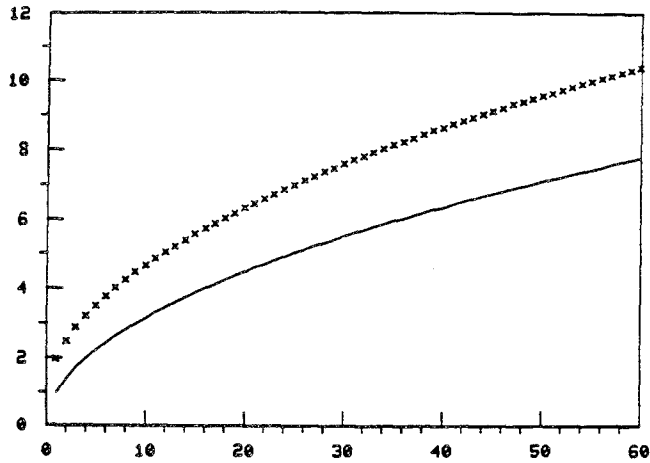
TITLE QUEUEING THEORY SERVER FUNCTION PHI(X)
 YLABEL Y
 XLABEL X
 X2LABEL MODEL-- $Y = B_0 + B_1 X^{B2}$
 X3LABEL JJF6*CS9.NONLINEAR6 JJF6*DATA.SHIER2 11/76
 PLOT Y PRED US X

YLABEL RESIDUALS
 PLOT RES X



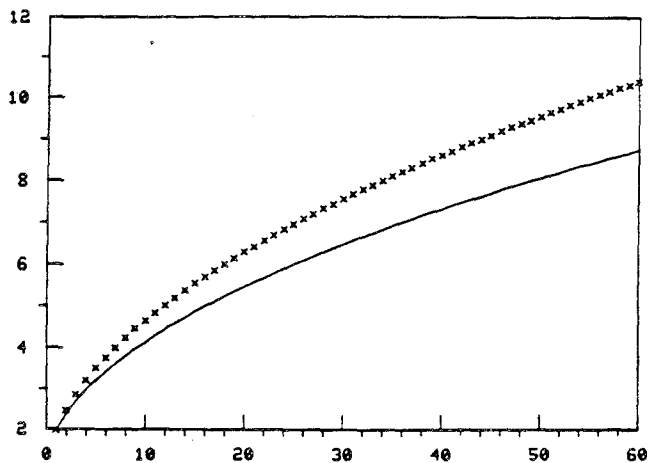
FULLY-SPECIFIED MODEL
 SAMPLE SIZE N = 60
 MODEL-- $Y = X^{.5}$
 NO REPLICATION CASE

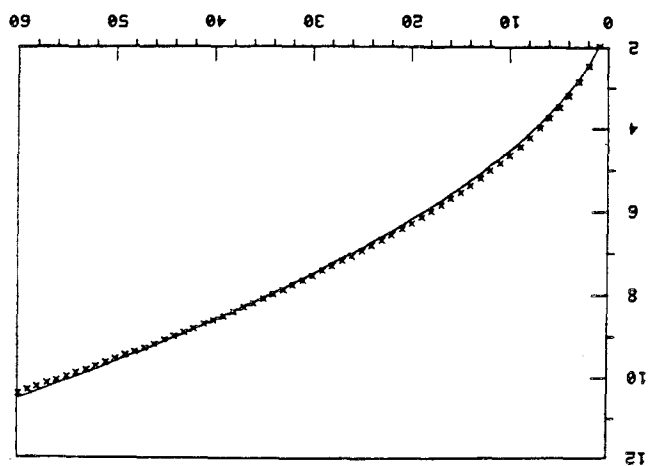
RESIDUAL STANDARD DEVIATION = 2.0607069731
 RESIDUAL DEGREES OF FREEDOM = 60



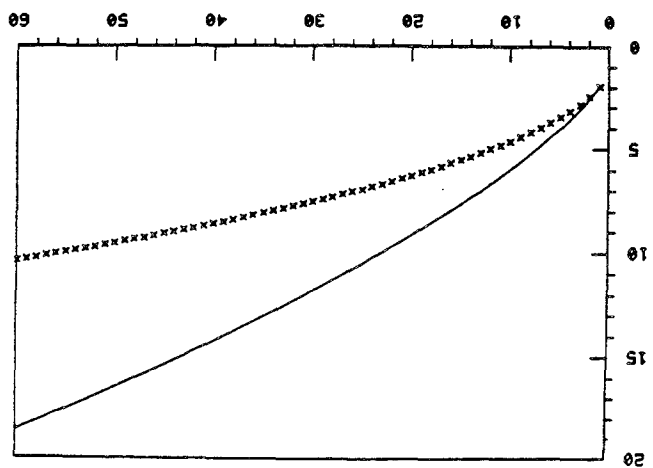
FULLY-SPECIFIED MODEL
 SAMPLE SIZE N = 60
 MODEL-- $Y = 1 + X^{.5}$
 NO REPLICATION CASE

RESIDUAL STANDARD DEVIATION = 1.1050422937
 RESIDUAL DEGREES OF FREEDOM = 60





FULLY-SPECIFIED MODEL
 SAMPLE SIZE N = 60
 MODEL--Y = 1 + XXX.55
 NO REPLICATION CASE
 RESIDUAL STANDARD DEVIATION = .0772325629
 RESIDUAL DEGREES OF FREEDOM = 60



FULLY-SPECIFIED MODEL
 SAMPLE SIZE N = 60
 MODEL--Y = 1+XXX.7
 NO REPLICATION CASE
 RESIDUAL STANDARD DEVIATION = 4.8758707643
 RESIDUAL DEGREES OF FREEDOM = 60

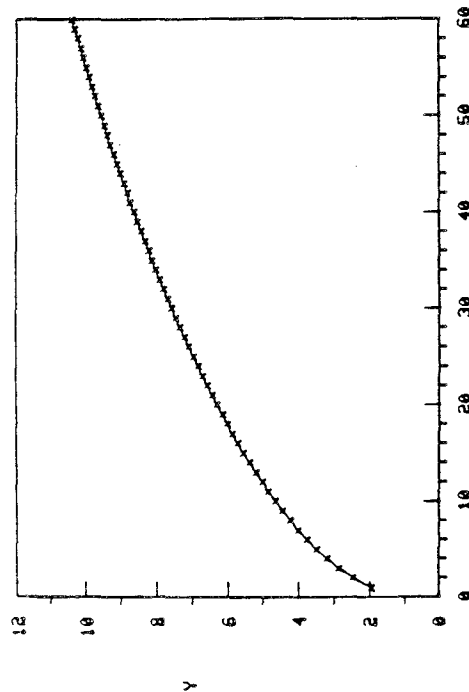
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 60
MODEL--Y = B0 + B1 * X**B2
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .1000-01 | .79239-01 | .10000+01 |
| 2-- | .5000-02 | .65638-01 | .85918+00 |
| 3-- | .2500-02 | .42829-02 | .11570+01 |
| 4-- | .1250-02 | .19770-02 | .78295+00 |
| 5-- | .62500-03 | .19752-02 | .12089+01 |
| | | | .77794+00 |
| | | | .12123+01 |
| | | | .77785+00 |
| | | | .12123+01 |
| | | | .55000+00 |
| | | | .51289+00 |
| | | | .50614+00 |
| | | | .50571+00 |
| | | | .50569+00 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 B0 .777848 (.2679-02)
2 B1 1.21235 (.1556-02)
3 B2 .505693 (.2539-03)

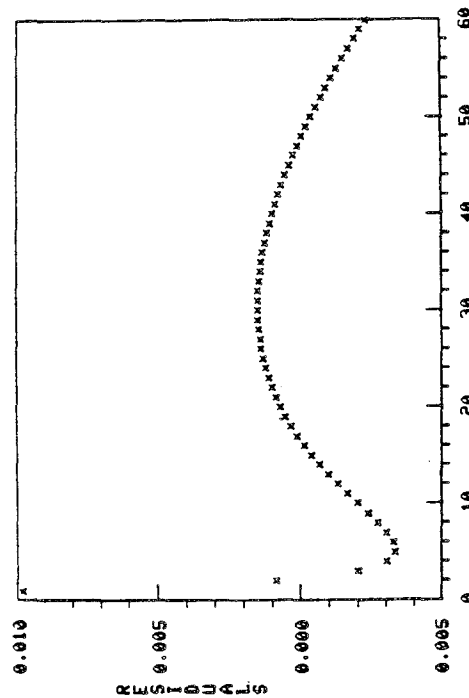
RESIDUAL STANDARD DEVIATION = .0019752173
DEGREES OF FREEDOM = 57

QUEUEING THEORY SERVER FUNCTION PHI(X)



MODEL--Y = B0 + B1 * X**B2
JJF6*CS9.NONLINEAR6 JJF6*DATA.SHIER2 11/76

QUEUEING THEORY SERVER FUNCTION PHI(X)



MODEL--Y = B0 + B1 * X**B2
JJF6*CS9.NONLINEAR6 JJF6*DATA.SHIER2 11/76

EXAMPLE 12

COMMENT EXAMPLE--LEW/WAMPLER CONCRETE STRENGTH
COMMENT MODEL --POWER IN DENOMINATOR

•
ECHO ON
HARDCOPY ON
BELL ON

•
SKIP 25
READ JJF6*DATA.LEW11 M Y
LET X=ALOG10(M-28)

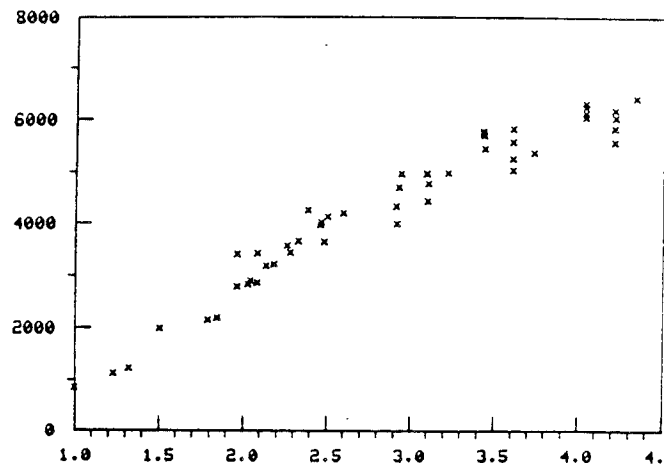
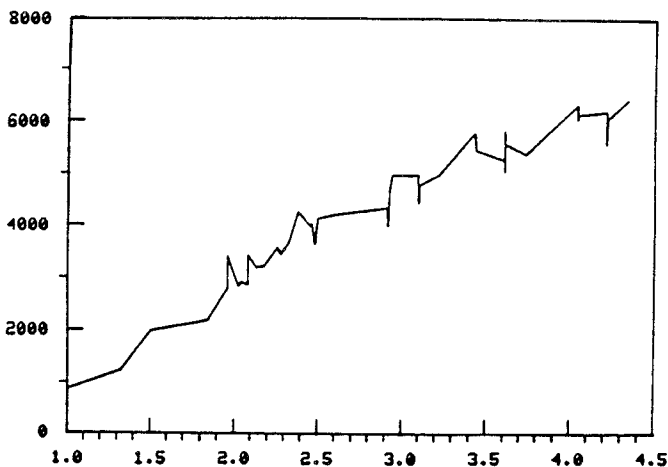
•
PLOT Y X

•
CHARACTERS X
LINES
PLOT Y X

•
LET K=9000
LET A=.001
LET B=-2
FIT Y=K/(1+K*A*X**B)

•
TITLE CONCRETE STRENGTH AS A FUNCTION OF MATURITY
YLABEL Y
XLABEL LOG10(MATURITY-28)
X2LABEL MODEL--Y = K / (1 + K*A*X**B) WITH X = LOG(M-28)
X3LABEL JJF6*CS9.NONLINEAR2 JJF6*DATA.LEW11 11/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED VS X

•
YLABEL RESIDUALS
PLOT RES X



LEAST SQUARES NON-LINEAR FIT

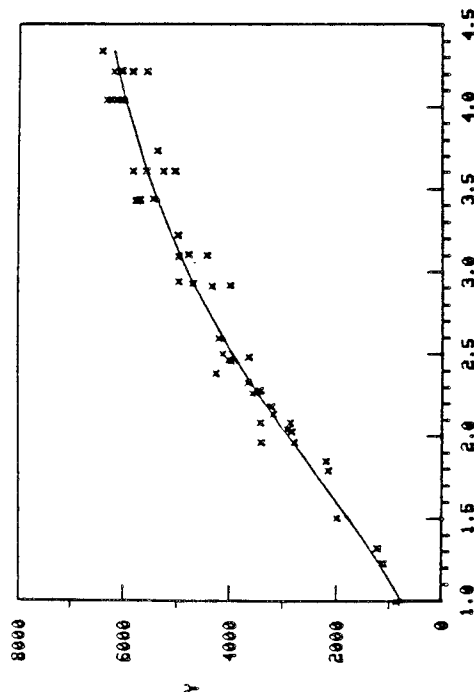
SAMPLE SIZE N = 50
 MODEL--Y=K/(1+K*XX**B)
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .4140350227+03
 REPLICATION DEGREES OF FREEDOM = 2
 NUMBER OF DISTINCT SUBSETS = 48

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .35234+03 | .90000+04 |
| 2-- | .22500-01 | .34590+03 | .10000-02 |
| 3-- | .11250-01 | .28200+03 | .73315+04 |
| 4-- | .56250-02 | .28200+03 | .10952-02 |
| | | | .77591+04 |
| | | | .11499-02 |
| | | | .77506+04 |
| | | | .11494-02 |
| | | | .24236+01 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 K 7750.42 (544.0)
 2 A .114950-02 (.1741-03)
 3 B -2.42372 (.2471)

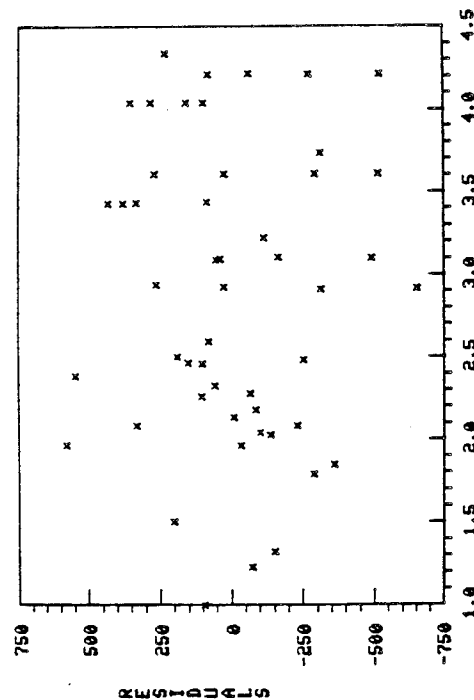
RESIDUAL STANDARD DEVIATION = 281.9972190857
 RESIDUAL DEGREES OF FREEDOM = 47
 REPLICATION STANDARD DEVIATION = 414.0350227356
 REPLICATION DEGREES OF FREEDOM = 2
 LACK OF FIT F RATIO = .4401 = THE 11.4769% POINT OF THE
 F DISTRIBUTION WITH 45 AND 2 DEGREES OF FREEDOM

CONCRETE STRENGTH AS A FUNCTION OF MATURITY



LOG10(MATURITY-28)
 MODEL--Y = K / (1 + K*XX**B) WITH X = LOG(M-28)
 JJF63CS9.NONLINEAR2 JJF63DATA.LEU11 11/76

CONCRETE STRENGTH AS A FUNCTION OF MATURITY



LOG10(MATURITY-28)
 MODEL--Y = K / (1 + K*XX**B) WITH X = LOG(M-28)
 JJF63CS9.NONLINEAR2 JJF63DATA.LEU11 11/76

EXAMPLE 13

COMMENT EXAMPLE--DUARIKA MISRA DENTAL RESEARCH STUDY
 COMMENT MODEL --EXPONENTIAL, QUADRATIC, SQUARE ROOT, RECIPROCAL
 COMMENT NOTE --COMPARING 4 NON-LINEAR MODLES

• ECHO ON
 • HARDCOPY ON
 • BELL ON

• READ JJF61DATA.MISRA1 U P

• CHARACTERS X
 • LINES
 • PLOT U P

• LET UM=400
 • LET C=.0003
 • FIT U = UM * (1 - EXP(-C*P))

• TITLE DENTAL RESEARCH MONOMOLECULAR ADSORPTION STUDY

YLABEL VOLUME

XLABEL PRESSURE

X2LABEL MODEL--U = UM * (1 - EXP(-C*P))

X3LABEL JJF61CS9.NONLINEAR29 JJF61DATA.MISRA1 4/78

CHARACTERS X BLANK

LINES BLANK SOLID

PLOT U PRED US P

LET S1=RESSD

• FIT U = UM * (1 - ((1+C*P/2)**(-2)))

LET S2=RESSD

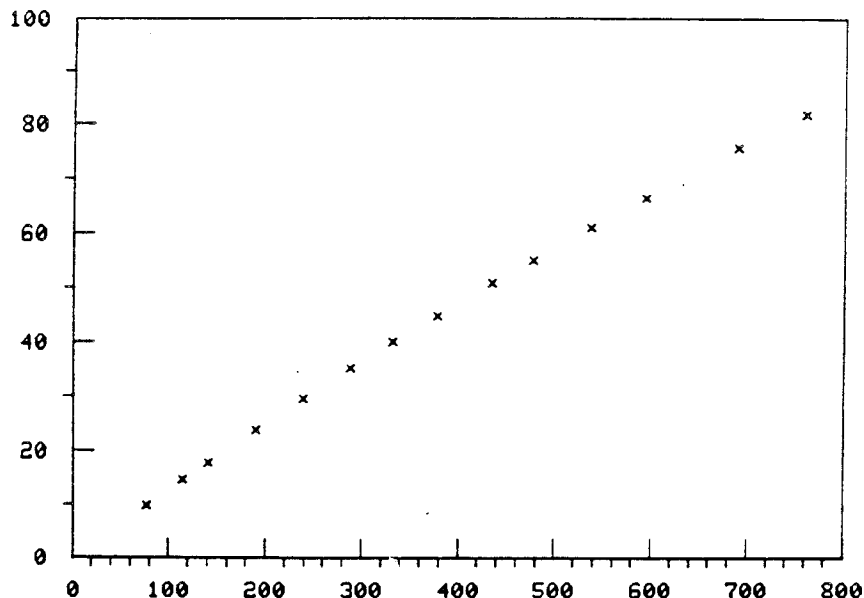
• FIT U = UM * (1 - ((1+2*C*P)**(-.5)))

LET S3=RESSD

• FIT U = UM*C*P*((1+C*P)**(-1))

LET S4=RESSD

• PRINT S1 S2 S3 S4

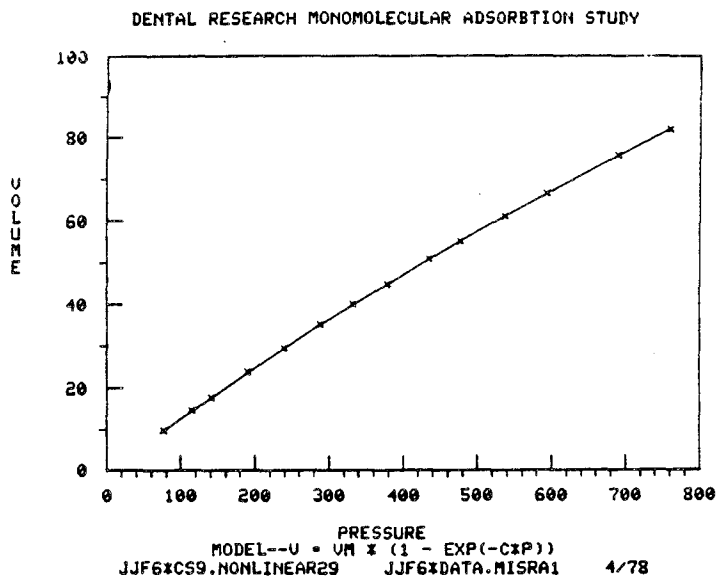


LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 14
 MODEL-- $U = U_M * (1 - \exp(-C * P))$
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .16396+01 * | .40000+03 .30000-03 |
| 2-- | .50625-01 | .99416+00 * | .36934+03 .33323-03 |
| 3-- | .25312-01 | .85428+00 * | .34023+03 .36523-03 |
| 4-- | .56953-01 | .68782+00 * | .32051+03 .39177-03 |
| 5-- | .28477-01 | .57803+00 * | .30338+03 .41701-03 |
| 6-- | .42715-01 | .52336+00 * | .28080+03 .45354-03 |
| 7-- | .21357-01 | .33959+00 * | .26578+03 .48445-03 |
| 8-- | .32036-01 | .24685+00 * | .25156+03 .51618-03 |
| 9-- | .16018-01 | .12916+00 * | .24493+03 .53399-03 |
| 10-- | .80090-02 | .10551+00 * | .23995+03 .54716-03 |
| 11-- | .40045-02 | .10188+00 * | .23900+03 .54999-03 |
| 12-- | .20023-02 | .10188+00 * | .23894+03 .55016-03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 U_M 238.940 (2.681)
 2 C .550162-03 (.7208-05)

RESIDUAL STANDARD DEVIATION = .1018779697
 RESIDUAL DEGREES OF FREEDOM = 12



LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 14
MODEL-- U = UM * (1-(1+C*P/2))**(-2))
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .31704+01 | .23894+03 |
| 2-- | .33750-01 | .20920+01 | .29097+03 |
| 3-- | .16875-01 | .31091+00 | .31733+03 |
| 4-- | .84375-02 | .16555+00 | .33322+03 |
| 5-- | .42187-02 | .79861-01 | .33765+03 |
| 6-- | .21094-02 | .79303-01 | .33798+03 |
| 7-- | .10547-02 | .79303-01 | .33799+03 |
| 8-- | .40045-02 | .79302-01 | .33799+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 UM 337.993 (3.136)
2 C .390396-03 (.4222-05)

RESIDUAL STANDARD DEVIATION = .0793014774
DEGREES OF FREEDOM = 12

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 14
MODEL-- U = UM*C*P*((1+C*P)**(-1))
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .26235+01 | .63642+03 |
| 2-- | .22500-01 | .23233+01 | .49492+03 |
| 3-- | .11250-01 | .56705+00 | .45727+03 |
| 4-- | .56250-02 | .13903+00 | .43904+03 |
| 5-- | .28125-02 | .68577-01 | .43744+03 |
| 6-- | .14062-02 | .68568-01 | .43737+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 UM 437.365 (3.617)
2 C .302277-03 (.2912-05)

RESIDUAL STANDARD DEVIATION = .0685681803
DEGREES OF FREEDOM = 12

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 14
MODEL-- U = UM * (1-(1+2*C*P)**(-.5))
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .59889+01 | .33799+03 |
| 2-- | .50625-01 | .39276+01 | .44754+03 |
| 3-- | .25312-01 | .86068+00 | .51803+03 |
| 4-- | .37969-01 | .58037+00 | .56860+03 |
| 5-- | .18984-01 | .20499+00 | .59791+03 |
| 6-- | .94922-02 | .14231+00 | .62522+03 |
| 7-- | .47461-02 | .60271-01 | .63533+03 |
| 8-- | .23730-02 | .58430-01 | .63639+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 UM 636.420 (4.623)
2 C .208139-03 (.1760-05)

RESIDUAL STANDARD DEVIATION = .0584296780
DEGREES OF FREEDOM = 12

PARAMETERS AND CONSTANTS--

S1 -- .1018780+00
S2 -- .7930148-01
S3 -- .5842968-01
S4 -- .6856818-01

EXAMPLE 14

COMMENT EXAMPLE--LARRY ROSZMAN QUANTUM DEFECTS FOR SULFUR I ATOM
 COMMENT MODEL --ARCTANGENT
 COMMENT NOTE --FITTING A THEORETICAL MODEL

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6*DATA.ROSZMAN1 X T
 LET Q = X-SQRT(-109737.3/T)

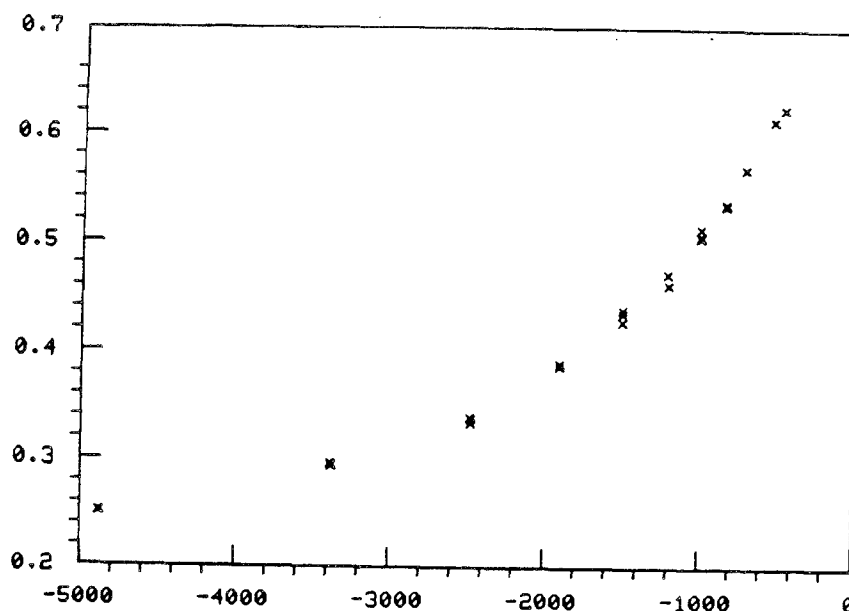
CHARACTERS X
 LINES BLANK
 PLOT Q T

LET A = .2
 LET B = -.00005
 LET C = 200
 LET D = -123
 FIT Q = A-B*T-ATAN(C/(T-D))/3.14159

TITLE QUANTUM DEFECTS FOR SULFUR I ATOM
 YLABEL QUANTUM DEFECT
 XLABEL EXCITED STATE ENERGY
 X2LABEL MODEL--Q = A - B*T - ATAN(C/(T-D))/3.14159
 X3LABEL JJF6*CS9.NONLINEAR45 DATA.ROSZMAN1 11/17/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Q PRED US T

PLOT Q T AND
 PLOT Q = A-B*T-ATAN(C/(T-D))/3.14159 FOR T = -5000 50 -400

YLABEL RESIDUALS
 PLOT RES T



LEAST SQUARES NON-LINEAR FIT 25
 SAMPLE SIZE N =
 MODEL-- Q - A - B*T - ATAN(C/(T-D))/3.14159
 NO. REPLICATION CASE

ITERATION CONVERGENCE RESIDUAL STANDARD DEVIATION
 NUMBER MEASURE MEASURE ESTIMATES

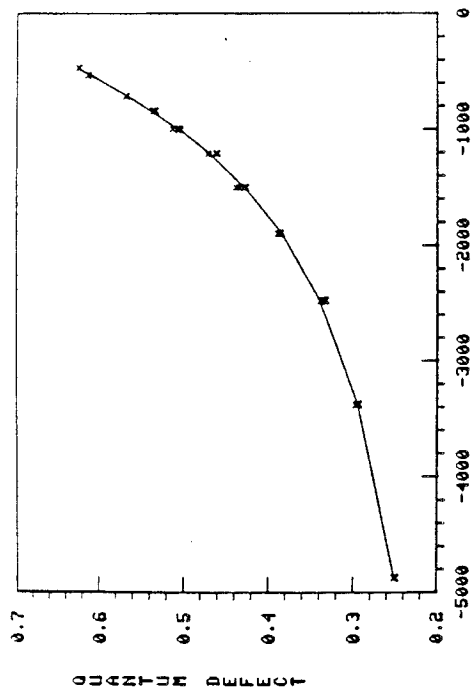
| | | | | | |
|---|-----------|-----------|-----------|------------|-------------|
| 1 | .1000E-01 | .29229+00 | .2000E+00 | -.5000E-04 | -.12300E+03 |
| 2 | .1500E-02 | .64731-01 | .29385+00 | -.19971-04 | -.48209E+03 |
| 3 | .2500E-02 | .49325-01 | .14199+00 | -.48900-06 | -.17000E+03 |
| 4 | .12556-01 | .35339-01 | .22996+00 | -.73756-05 | -.21816E+03 |
| 5 | .63281-02 | .74600-02 | .23390+00 | -.10476-04 | -.12025E+03 |
| 6 | .31641-02 | .51891-02 | .20577+00 | -.67880-05 | -.15801E+03 |
| 7 | .15820-02 | .48543-02 | .20194+00 | -.61932-05 | -.18133E+03 |
| 8 | .79102-03 | .48542-02 | .20195+00 | -.61919-05 | -.18130E+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

| | | |
|-----|-------------|------------|
| 1 A | .201944 | (.1927-01) |
| 2 B | -.619161-05 | (.3216-05) |
| 3 C | 1204.56 | (74.64) |
| 4 D | -181.384 | (49.88) |

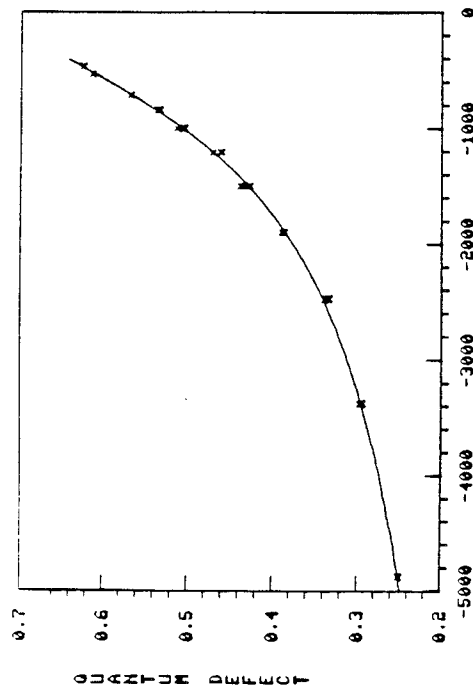
RESIDUAL STANDARD DEVIATION : .0048542210
 DEGREES OF FREEDOM : 21

QUANTUM DEFECTS FOR SULFUR I ATOM



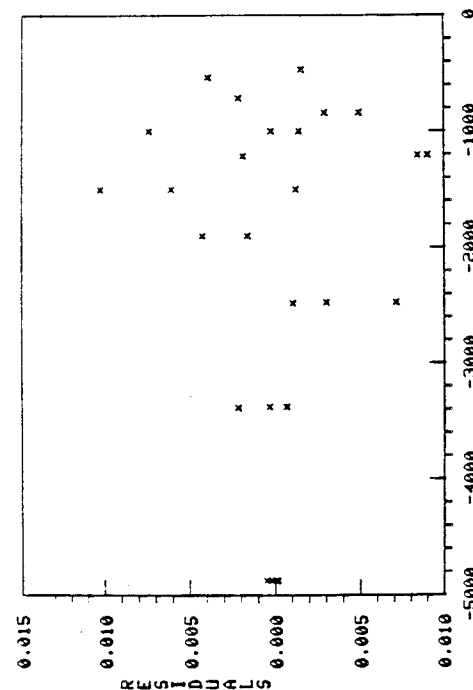
EXCITED STATE ENERGY
 MODEL-- Q - A - B*T - ATAN(C/(T-D))/3.14159
 JJF6KCS9.NONLINEAR45 DATA.R05ZMAN1 11/17/78

QUANTUM DEFECTS FOR SULFUR I ATOM



EXCITED STATE ENERGY
 MODEL-- Q - A - B*T - ATAN(C/(T-D))/3.14159
 JJF6KCS9.NONLINEAR45 DATA.R05ZMAN1 11/17/78

QUANTUM DEFECTS FOR SULFUR I ATOM



EXCITED STATE ENERGY
 MODEL-- Q - A - B*T - ATAN(C/(T-D))/3.14159
 JJF6KCS9.NONLINEAR45 DATA.R05ZMAN1 11/17/78

EXAMPLE 14

COMMENT EXAMPLE--LARRY ROSZMAN QUANTUM DEFECTS FOR SULFUR I ATOM
 COMMENT MODEL --ARCTANGENT
 COMMENT NOTE --FITTING A THEORETICAL MODEL

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6*DATA.ROSZMAN1 X T
 LET Q = X-SQRT(-109737.3/T)

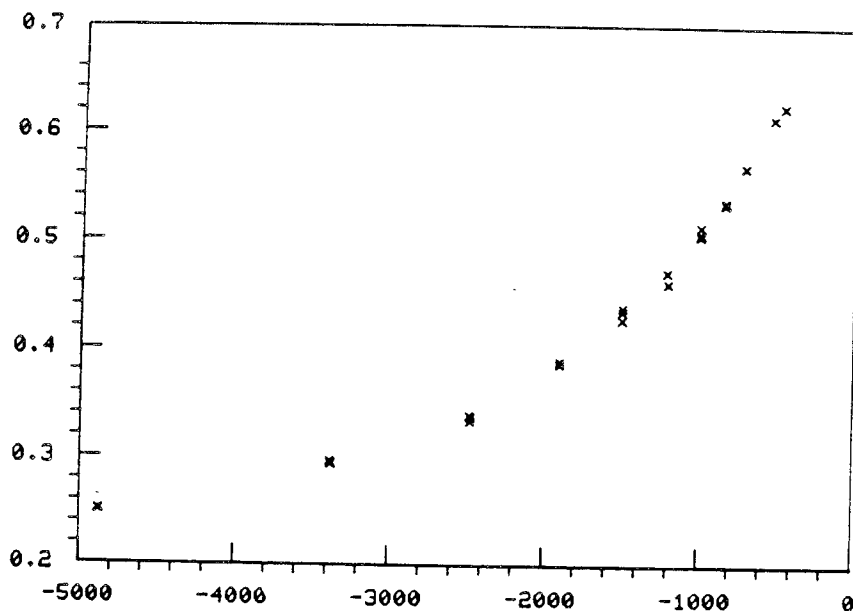
CHARACTERS X
 LINES BLANK
 PLOT Q T

LET A = .2
 LET B = -.00005
 LET C = 200
 LET D = -123
 FIT Q = A-B*T-ATAN(C/(T-D))/3.14159

TITLE QUANTUM DEFECTS FOR SULFUR I ATOM
 YLABEL QUANTUM DEFECT
 XLABEL EXCITED STATE ENERGY
 X2LABEL MODEL--Q = A - B*T - ATAN(C/(T-D))/3.14159
 X3LABEL JJF6*CS9.NONLINEAR45 DATA.ROSZMAN1 11/17/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Q PRED VS T

PLOT Q T AND
 PLOT Q = A-B*T-ATAN(C/(T-D))/3.14159 FOR T = -5000 50 -400

YLABEL RESIDUALS
 PLOT RES T



EXAMPLE 15

COMMENT-EXAMPLE--KEN ECKERLE TRANSMITTANCE STUDY
COMMENT MODEL --LORENTZIAN AND GAUSSIAN

ECHO ON
HARDCOPY ON
BELL ON

READ JJF6\$DATA.ECKERLE4 X Y

CHARACTERS X
LINES
PLOT Y X

VLIM -.1 .4
PLOT Y X

LET A = .3
LET MU1 = 450

LET B = 4
FIT Y = A / (1 + ((X - MU1) / B) ** 2)

TITLE CIRCULAR INTERFERENCE FILTER STUDY

VLABEL TRANSMITTANCE
XLABEL WAVELENGTH

X2LABEL MODEL--Y = A / (1 + ((X - MU1) / B) ** 2)

X3LABEL JJF6\$CS9.NONLINEAR26 JJF6\$DATA.ECKERLE4 4/20/78

CHARACTERS X BLANK
LINES BLANK SOLID

PLOT Y PRED US X
LET PRED1 = PRED

LET S1 = RESSD

LET D = 1

LET E = 10

LET F = .4

LET MU2 = 450

[FIT Y = (D/E) * F * EXP(-0.5 * ((X - MU2) / E) ** 2)]

X2LABEL MODEL--Y = (D/E) * F * EXP(-0.5 * ((X - MU2) / E) ** 2)

LINES BLANK DOTTED

PLOT Y PRED US X

LET PRED2 = PRED

LET S2 = RESSD

X2LABEL SOLID = LORENTZIAN; DOTTED = GAUSSIAN

CHARACTERS X BLANK BLANK

LINES BLANK SOLID DOT

PLOT Y PRED1 PRED2 US X

LET MN = MIN(X)

LET MX = MAX(X)

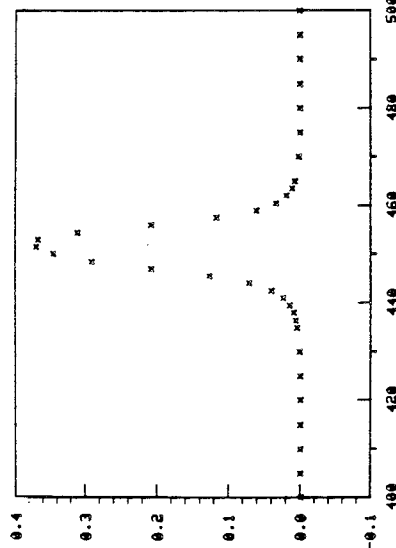
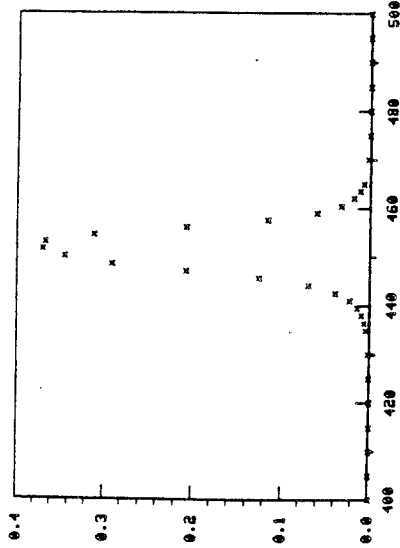
LET INC = (MX - MN) / 100

PLOT Y X AND

PLOT Y = A / (1 + ((X - MU1) / B) ** 2) FOR X = MN INC MX AND

PLOT Y = (D/E) * F * EXP(-0.5 * ((X - MU2) / E) ** 2) FOR X = MN INC MX

PRINT A B MU1 D E F MU2 S1 S2



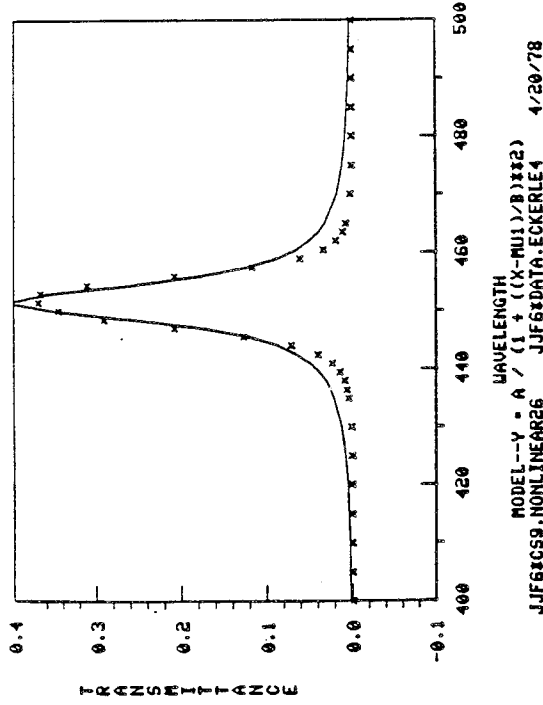
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 35
MODEL--Y = A/(1+(X-MU1)/B)**2)
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .55012-01 | .30000+00 |
| 2-- | .50000-02 | .52846-01 | .49220+00 |
| 3-- | .19222+00 | .48949-01 | .27900+00 |
| 4-- | .96108-01 | .26227-01 | .37642+00 |
| 5-- | .48054-01 | .24440-01 | .43592+00 |
| 6-- | .24027-01 | .23333-01 | .45175+03 |
| 7-- | .12014-01 | .22214-01 | .45147+03 |
| 8-- | .60068-02 | .22115-01 | .45152+03 |
| 9-- | .30034-02 | .22001-01 | .45156+03 |
| 10-- | .17319+00 | .21999-01 | .45155+03 |
| 11-- | .86595-01 | .21998-01 | .45155+03 |
| 12-- | .43298-01 | .21996-01 | .45155+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 A .408500 (.1979-01)
2 MU1 451.547 (.2215)
3 B 3.93598 (.2589)

RESIDUAL STANDARD DEVIATION = .0219961645
DEGREES OF FREEDOM = 32

CIRCULAR INTERFERENCE FILTER STUDY



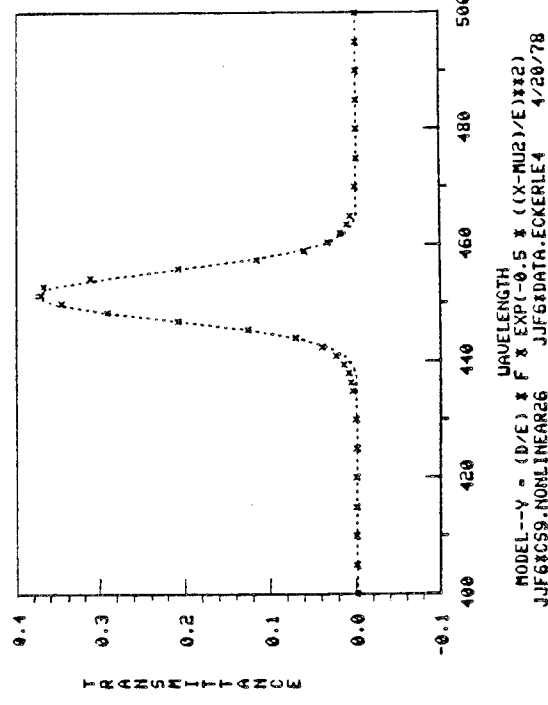
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 35
MODEL--Y = (D/E) * F * EXP(-0.5 * ((X-MU2)/E)**2)
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|--|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .13025+00 | .10000+01 |
| 2-- | .29193+01 | .85087-01 | .13696+01 |
| PARAMETER 3 IS LINEARLY DEPENDANT UPON PREVIOUS PARAMETERS | | | |
| 3-- | .29193+01 | .85087-01 | .13696+01 |
| 4-- | .14596+01 | .74460-01 | .15230+01 |
| 5-- | .72982+00 | .45686-01 | .19160+01 |
| 6-- | .36491+00 | .19652-01 | .24331+01 |
| 7-- | .18246+00 | .83489-02 | .27378+01 |
| 8-- | .91238-01 | .69109-02 | .28267+01 |
| 9-- | .45614-01 | .69051-02 | .28375+01 |
| 10-- | .22807-01 | .69041-02 | .28380+01 |
| PARAMETER WILL BE KEPT CONSTANT | | | |
| 3-- | .29193+01 | .85087-01 | .13696+01 |
| 4-- | .14596+01 | .74460-01 | .15230+01 |
| 5-- | .72982+00 | .45686-01 | .19160+01 |
| 6-- | .36491+00 | .19652-01 | .24331+01 |
| 7-- | .18246+00 | .83489-02 | .27378+01 |
| 8-- | .91238-01 | .69109-02 | .28267+01 |
| 9-- | .45614-01 | .69051-02 | .28375+01 |
| 10-- | .22807-01 | .69041-02 | .28380+01 |

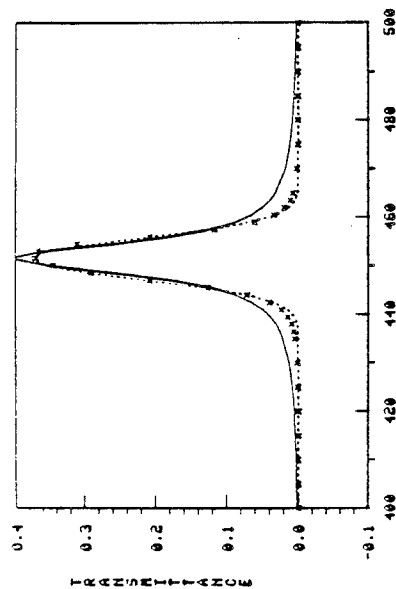
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 D 2.83794 (.2837-01)
2 E 4.09056 (.5387-01)
3 F .547828
4 MU2 451.515 (.6164-01)

RESIDUAL STANDARD DEVIATION = .00659040230
DEGREES OF FREEDOM = 31

CIRCULAR INTERFERENCE FILTER STUDY

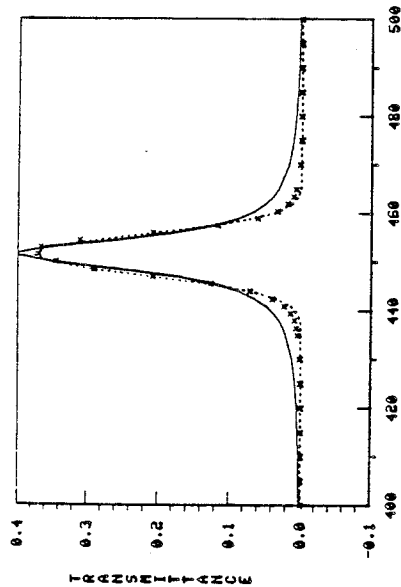


CIRCULAR INTERFERENCE FILTER STUDY



SOLID - LORENTZIAN; DOTTED - GAUSSIAN
JJF61C59.NONLINEAR26 JJF61DATA.ECKERLE4 4/20/78

CIRCULAR INTERFERENCE FILTER STUDY



SOLID - LORENTZIAN; DOTTED - GAUSSIAN
JJF61C59.NONLINEAR26 JJF61DATA.ECKERLE4 4/20/78

PARAMETERS AND CONSTANTS--

| | | |
|-----|----|-------------|
| A | -- | .4084996+00 |
| B | -- | .3935978+01 |
| MU1 | -- | .4515468+03 |
| D | -- | .2837935+01 |
| E | -- | .4090563+01 |
| F | -- | .5478279+00 |
| MU2 | -- | .4515153+03 |
| S1 | -- | .2199616-01 |
| S2 | -- | .6904023-02 |

COMMENT EXAMPLE--FINDING THE ULTRASONIC CALIBRATION CURVE
 COMMENT MODEL --PRODUCT OF TWO BESSEL FUNCTIONS

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6*DATA.HIGGINS1
 NAME UOUT 1
 NAME UAPP 3

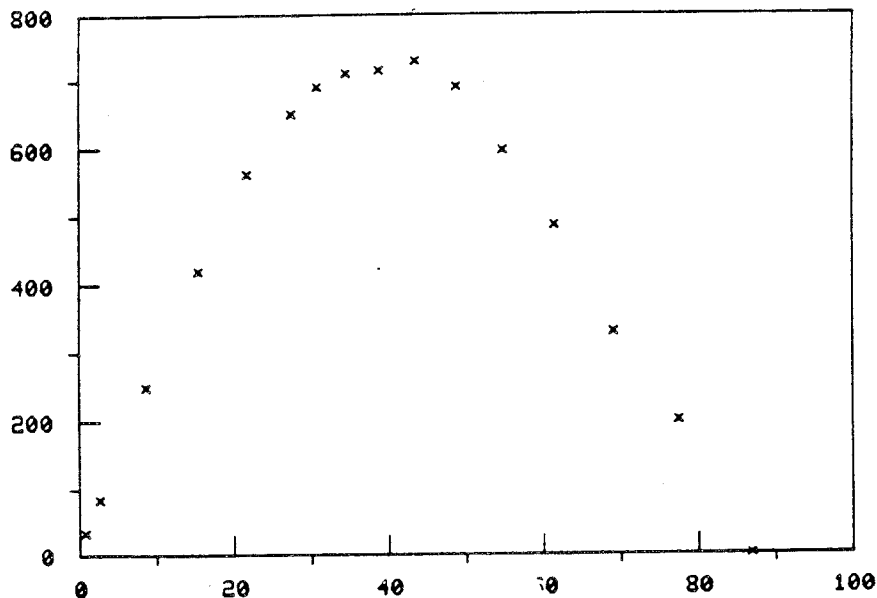
CHARACTERS X
 LINES
 PLOT UOUT UAPP

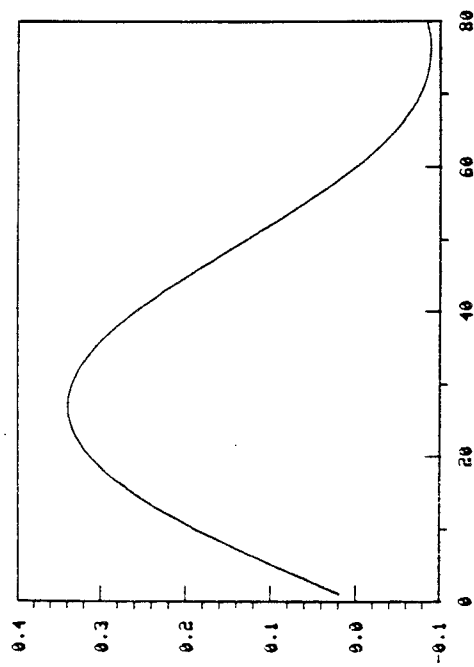
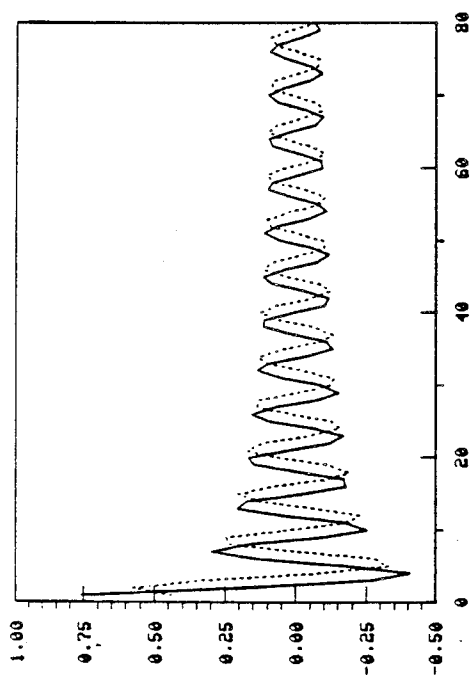
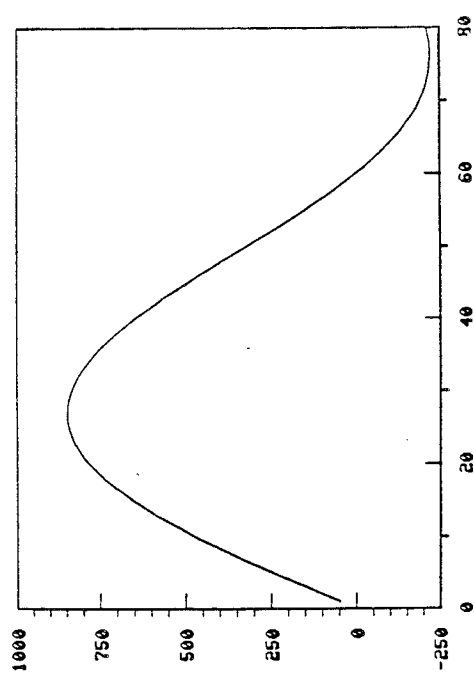
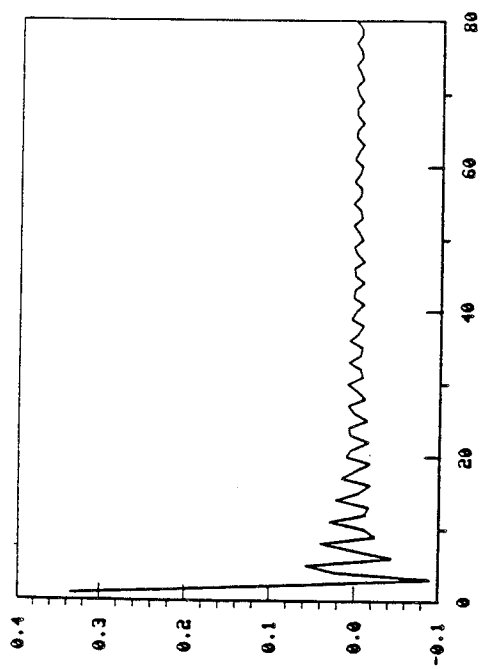
CHARACTERS
 LINES SOLID DOTTED
 PLOT Y = BESS0(X) FOR X = 1 1 80 AND
 PLOT Y = BESS1(X) FOR X = 1 1 80
 PLOT Y = BESS0(X)*BESS1(X) FOR X = 1 1 80
 PLOT Y = BESS0(.04*X)*BESS1(.04*X) FOR X = 1 1 80
 PLOT Y = 2500*BESS0(.04*X)*BESS1(.04*X) FOR X = 1 1 80

LET A0 = 2500
 LET A1 = .04
 FIT UOUT = A0 * BESS0(A1*UAPP) * BESS1(A1*UAPP)

TITLE ULTRASONOVISION CALIBRATION
 YLABEL VOLTAGE OUT
 XLABEL VOLTAGE APPLIED
 X2LABEL MODEL--Y = A0 * BESS0(A1*UAPP) * BESS1(A1*UAPP)
 X3LABEL JJF6*CS9.NONLINEAR27 JJF6*DATA.HIGGINS1 2/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT UOUT PRED VS UAPP

YLABEL RESIDUALS
 PLOT RES UAPP



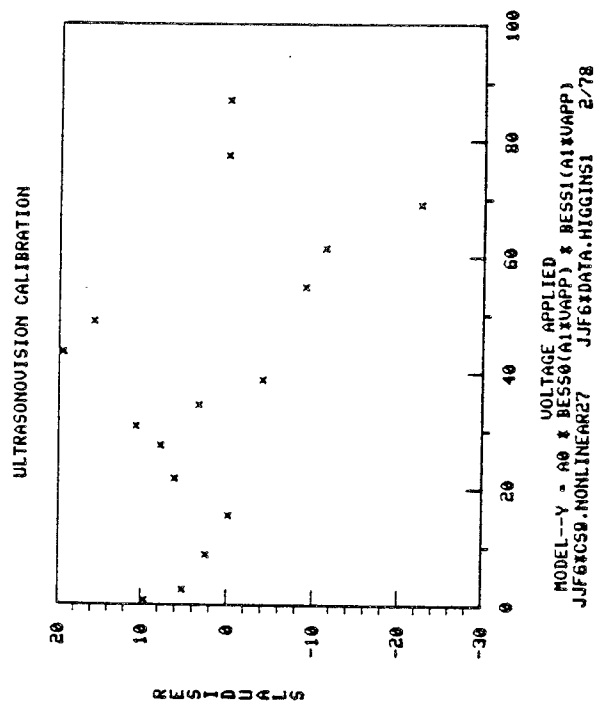
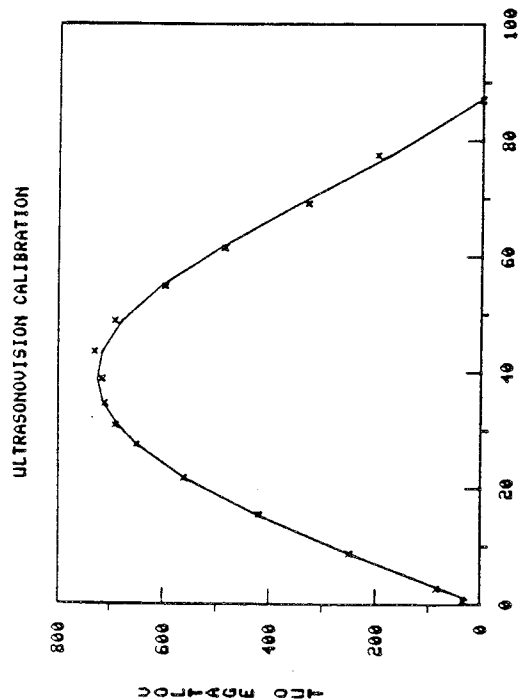


LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 16
 MODEL-- VOUT = A0 * BESS0(A1XUAPP) * BESS1(A1XUAPP)
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
| 1-- | .10000-01 | .30272+03 | .25000+04 |
| 2-- | .50000-02 | .13256+03 | .17658+04 |
| 3-- | .25000-02 | .25082+02 | .21089+04 |
| 4-- | .12500-02 | .11043+02 | .21369+04 |
| 5-- | .62500-03 | .11036+02 | .21382+04 |
| | | | .40000-01 |
| | | | .30485-01 |
| | | | .26768-01 |
| | | | .27559-01 |
| | | | .27554-01 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 A0 2138.24 (11.75)
 2 A1 .275540-01 (.1038-03)

RESIDUAL STANDARD DEVIATION = 11.0361322165
 RESIDUAL DEGREES OF FREEDOM = 14



EXAMPLE 17

COMMENT EXAMPLE---DAN CHUIRUT ULTRASONIC REFERENCE BLOCK ANALYSIS
 COMMENT MODEL --EXPONENTIAL/LINEAR
 COMMENT NOTE --AUTOMATIC CHECK FOR REPLICATION

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6XDATA.CHUIRUT1 Y X

PLOT Y X
 PLOT Y X X
 CHARACTERS X
 PLOT Y X X

LET ALPHA=.15
 LET A=.004
 LET B=.01
 FIT Y = EXP(-ALPHA*Y)/(A+B*Y)

TITLE ULTRASONIC REFERENCE BLOCK STUDY

VLABEL ULTRASONIC RESPONSE

XLABEL METAL DISTANCE

X2LABEL MODEL--Y = EXP(-ALPHA*Y) / (A+B*Y)

X3LABEL JJF6XCS9.NONLINEAR25 JJF6XDATA.CHUIRUT1

1/78

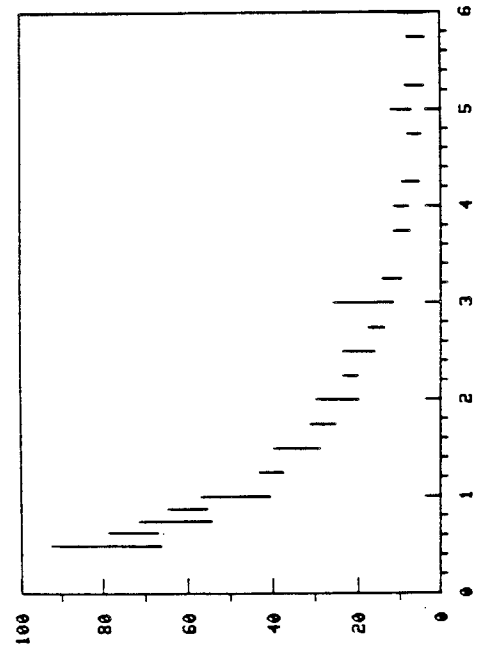
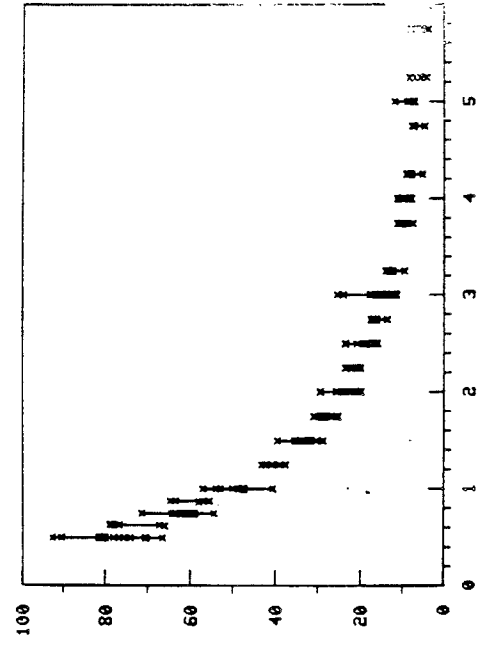
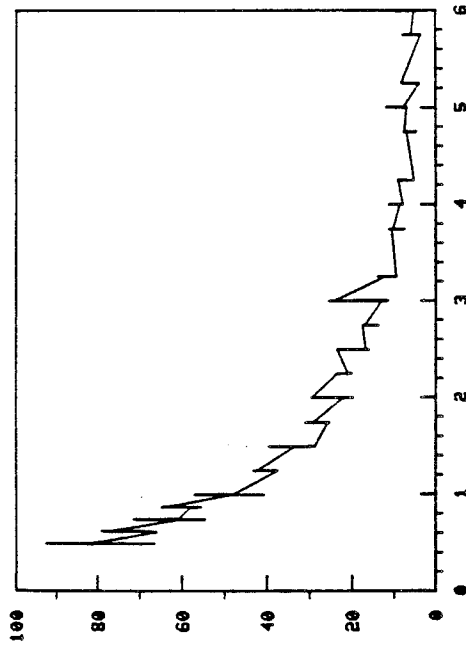
CHARACTERS X BLANK

LINES BLANK SOLID

PLOT Y PRED US X

VLABEL RESIDUALS

PLOT RES X



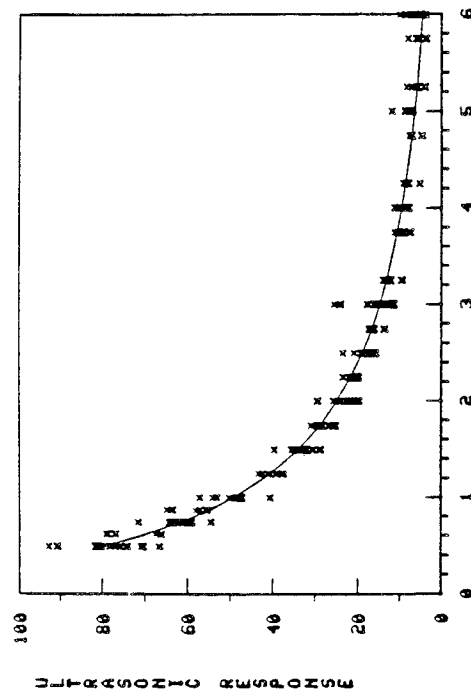
LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 214
 MODEL--Y = EXP(-ALPHA*XX)/(A+B*XX)
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .3237755179+01
 REPLICATION DEGREES OF FREEDOM = 190
 NUMBER OF DISTINCT SUBSETS = 24

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|-------------------------------|
| 1-- | .10000-01 | .10785+02 * | .15000+00 .40000-02 .10000-01 |
| 2-- | .50000-02 | .37239+01 * | .18067+00 .55545-02 .10719-01 |
| 3-- | .25000-02 | .33631+01 * | .19045+00 .61197-02 .10523-01 |
| 4-- | .12500-02 | .33628+01 * | .19035+00 .61344-02 .10528-01 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 ALPHA .190307 (.2208-01)
 2 A .613365-02 (.3495-03)
 3 B .105298-01 (.8033-03)

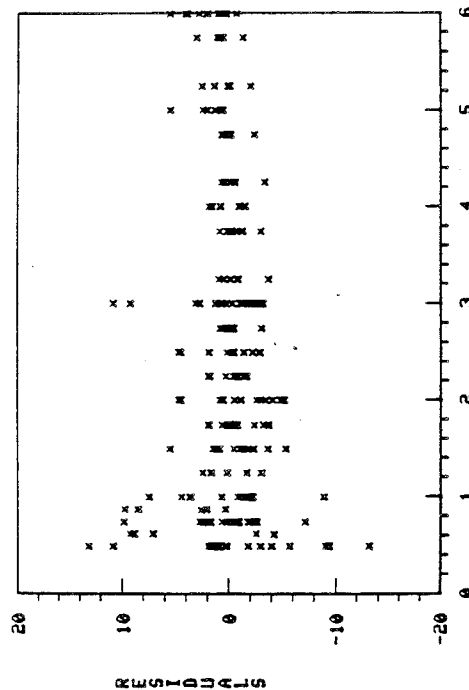
RESIDUAL STANDARD DEVIATION = 3.3627645671
 DEGREES OF FREEDOM = 211
 REPLICATION STANDARD DEVIATION = 3.2377551794
 REPLICATION DEGREES OF FREEDOM = 190
 LACK OF FIT F RATIO = 1.7909 - THE 97.8064% POINT OF THE
 F DISTRIBUTION WITH 21 AND 190 DEGREES OF FREEDOM

ULTRASONIC REFERENCE BLOCK STUDY



METAL DISTANCE
 MODEL--Y = EXP(-ALPHA*XX) / (A+B*XX)
 JJF6XCS9.NONLINEAR25 JJF6XDATA.CHUIRUT1 1/78

ULTRASONIC REFERENCE BLOCK STUDY



METAL DISTANCE
 MODEL--Y = EXP(-ALPHA*XX) / (A+B*XX)
 JJF6XCS9.NONLINEAR25 JJF6XDATA.CHUIRUT1 1/78

EXAMPLE 18

COMMENT EXAMPLE--H. S. LEW CONCRETE PULL-OUT BOND STRENGTH
COMMENT MODEL --LINEAR/LINEAR

ECHO ON
HARDCOPY ON
BELL ON

SKIP 10
READ JJF6*DATA.LEW4 STREN MAT

CHARACTERS X
LINES
PLOT STREN MAT

LET STREN2(1)=11
LET STREN2(2)=22
LET STREN2(3)=26
LET MAT2(1)=1.7
LET MAT2(2)=2.6
LET MAT2(3)=3.2
EXACT 1/1 RATIONAL FIT STREN2 MAT2 STREN MAT

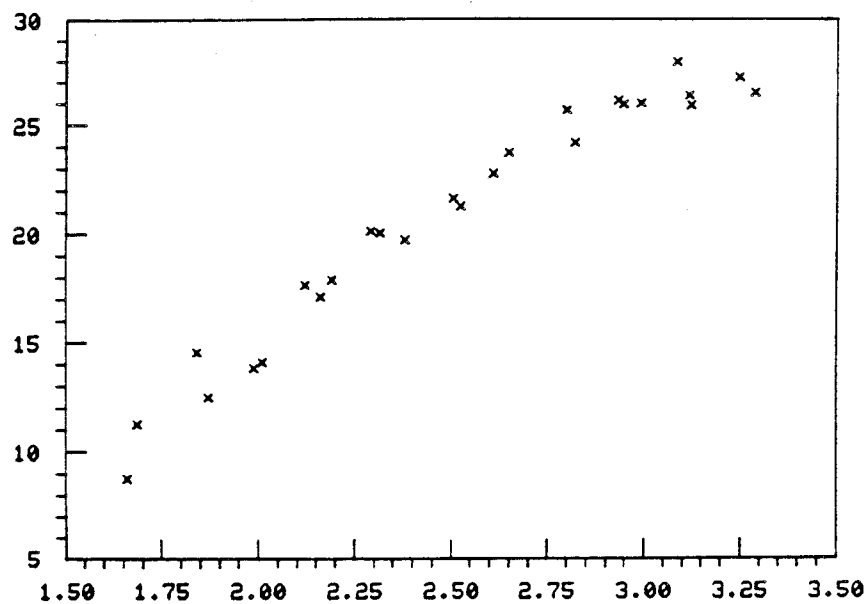
TITLE PULL-OUT BOND STRENGTH OF CONCRETE
YLABEL BOND STRENGTH
XLABEL MATURITY (= [TEMPERATURE(F) - 10] * ELAPSED DAYS)
X2LABEL EXACT LINEAR/LINEAR RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR39 JJF6*DATA.LEW4 1/76
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT STREN PRED US MAT

FIT STREN = (A0+A1*MAT)/(1+B1*MAT)

X2LABEL MODEL--STREN = (A0 + A1*MAT) / (1 + B1 * MAT)
PLOT STREN PRED US MAT

YLABEL RESIDUALS
PLOT RES MAT

XLABEL
NORMAL PROBABILITY PLOT RES



EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 3 |
| DEGREE OF NUMERATOR | = | 1 |
| DEGREE OF DENOMINATOR | = | 1 |

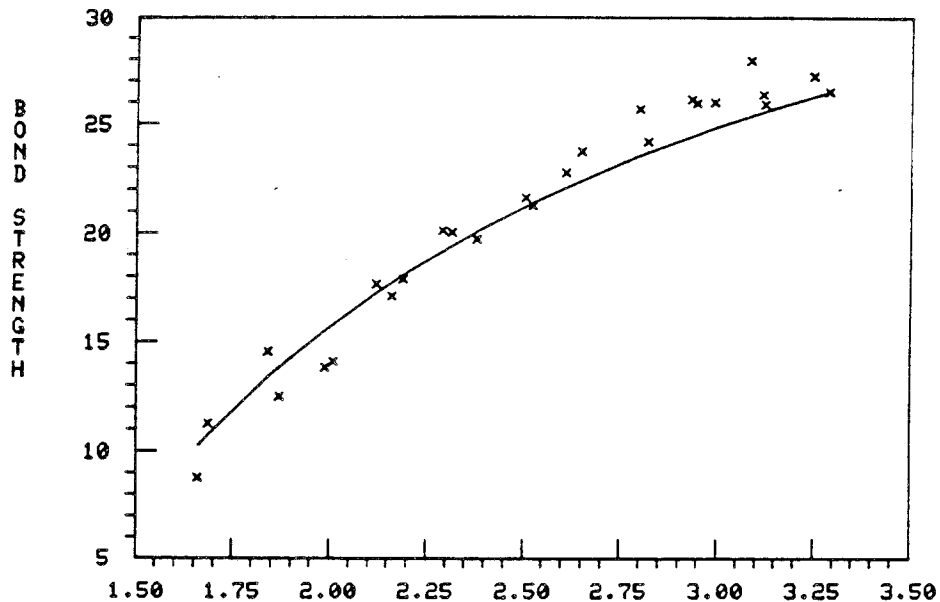
| | | | | |
|-----------------|----|---|---------------|--------------|
| NUMERATOR --A0 | A1 | = | -.55000179+03 | .44000142+03 |
| DENOMINATOR--B0 | B1 | = | .10000000+01 | .10000033+02 |

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|----|
| NUMBER OF POINTS IN SECOND SET | = | 26 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 3 |
| RESIDUAL DEGREES OF FREEDOM | = | 23 |

| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .35384189+02 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .12403399+01 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .96846149+00 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .25857794+01 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.17317243+01 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .25857794+01 |

PULL-OUT BOND STRENGTH OF CONCRETE

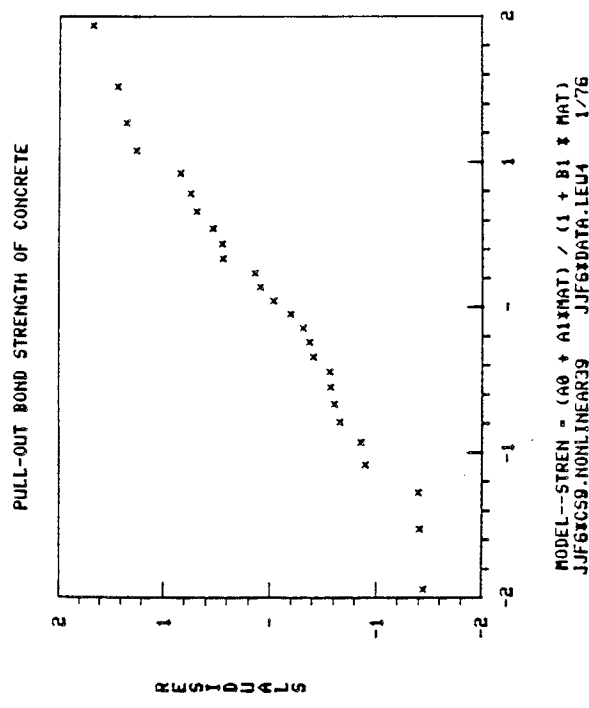
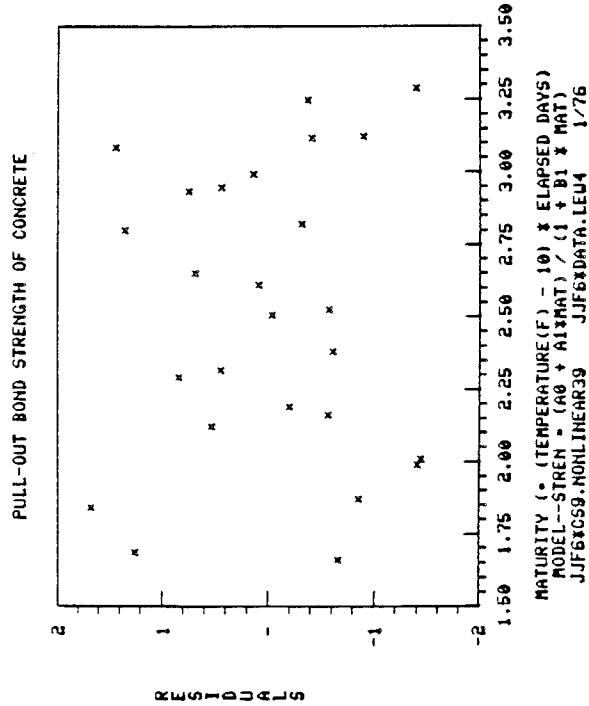
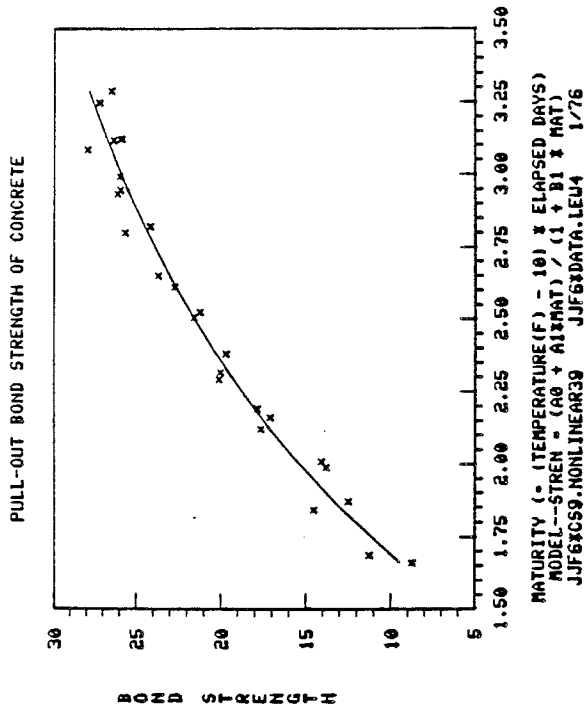


MATURITY (= (TEMPERATURE(F) - 10) * ELAPSED DAYS)
EXACT LINEAR/LINEAR RATIONAL FIT
JJF6*CS9.NONLINEAR39 JJF6*DATA.LEW4 1/76

LEAST SQUARES NON-LINEAR FIT 26
 SAMPLE SIZE N =
 MODEL--STREN = (A0+A1MAT)/(1+B1MAT)
 NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1-- | .10000-01 | .12403+01 | -.55000+03 |
| 2-- | .50000-02 | .94361+00 | -.57271+03 |
| 3-- | .35000-02 | .93787+00 | -.53361+03 |
| 4-- | .37500-02 | .93750+00 | -.48422+03 |
| 5-- | .18750-02 | .93678+00 | -.43264+03 |
| 6-- | .28125-02 | .93619+00 | -.35391+03 |
| 7-- | .14062-02 | .93478+00 | -.29228+03 |
| 8-- | .21094-02 | .93419+00 | -.22600+03 |
| 9-- | .10547-02 | .93282+00 | -.19828+03 |
| 10-- | .52734-03 | .93285+00 | -.18195+03 |
| 11-- | .26367-03 | .93253+00 | -.18161+03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
 1 A0 (-181.569)
 2 A1 (141.515)
 3 B1 (2.79461)
 RESIDUAL STANDARD DEVIATION = 2.79461
 DEGREES OF FREEDOM = 23

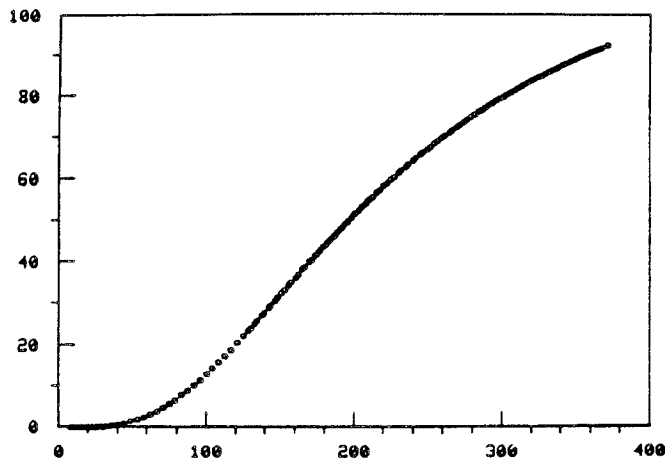


EXAMPLE 19

```

COMMENT EXAMPLE--DICK KIRBY SCANNING ELECTRON MICROSCOPE
COMMENT LINE SPACING STANDARDS
COMMENT MODEL --LINEAR/LINEAR AND QUADRATIC/QUADRATIC
.
ECHO ON
HARDCOPY ON
BELL ON
.
READ JJF61DATA.KIRBY2 X Y
.
CHARACTERS CIRCLE
LINES
PLOT Y X
.
LET ID = 01Y
LET ID(1)=1
LET ID(75)=1
LET ID(152)=1
LET Y2=Y SUBSET ID 1
LET X2=X SUBSET ID 1
PACK Y2 X2 SUBSET ID 1
EXACT 1/1 RATIONAL FIT Y2 X2 Y X
.
TITLE SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS
YLABEL Y
XLABEL X
X2LABEL EXACT FIT THROUGH 3 POINTS OF LINEAR/LINEAR MODEL
X3LABEL JJF61CS9.NONLINEAR11 JJF61DATA.KIRBY2 11/9/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT Y PRED US X
.
FIT Y = (A0+A1X)/(1+B1X)
.
X2LABEL MODEL--Y = (A0 + A1X) / (1 + B1X)
PLOT Y PRED US X
.
YLABEL RESIDUALS
PLOT RES X
.
LET ID(30)=1
LET ID(110)=1
LET Y2=Y SUBSET ID 1
LET X2=X SUBSET ID 1
PACK Y2 X2 SUBSET ID 1
EXACT 2/2 RATIONAL FIT Y2 X2 Y X
.
YLABEL Y
X2LABEL EXACT FIT THROUGH 5 POINTS OF QUADRATIC/QUADRATIC MODEL
PLOT Y PRED US X
.
FIT Y = (A0+A1X+A2X1X)/(1+B1X+B2X1X)
.
X2LABEL MODEL--Y = (A0 + A1X + A2X1X) / (1 + B1X + B2X1X)
PLOT Y PRED US X
.
YLABEL RESIDUALS
PLOT RES X

```



EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 3 |
| DEGREE OF NUMERATOR | = | 1 |
| DEGREE OF DENOMINATOR | = | 1 |

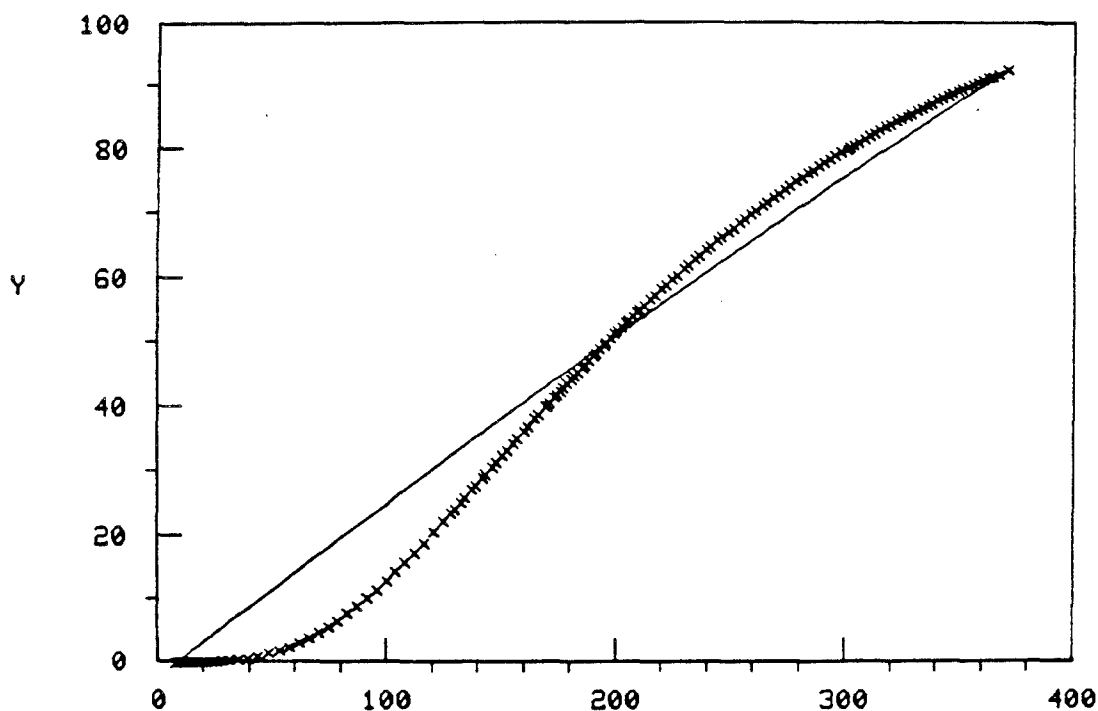
| | | | | |
|-----------------|----|---|---------------|--------------|
| NUMERATOR --A0 | A1 | = | -.23937376+01 | .27869345+00 |
| DENOMINATOR--B0 | B1 | = | .10000000+01 | .25954249-03 |

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|-----|
| NUMBER OF POINTS IN SECOND SET | = | 152 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 3 |
| RESIDUAL DEGREES OF FREEDOM | = | 149 |

| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .45225974+04 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .55093559+01 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .42797687+01 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .43819942+01 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.12615372+02 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .12615372+02 |

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



EXACT FIT THROUGH 3 POINTS OF LINEAR/LINEAR MODEL
JJF6*CS9.NONLINEAR11 JJF6*DATA.KIRBY2 11/9/78

LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 152
 MODEL--Y = (A0+A1X)/(1+B1X)
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .1202078687-01
 REPLICATION DEGREES OF FREEDOM = 1
 NUMBER OF DISTINCT SUBSETS = 151

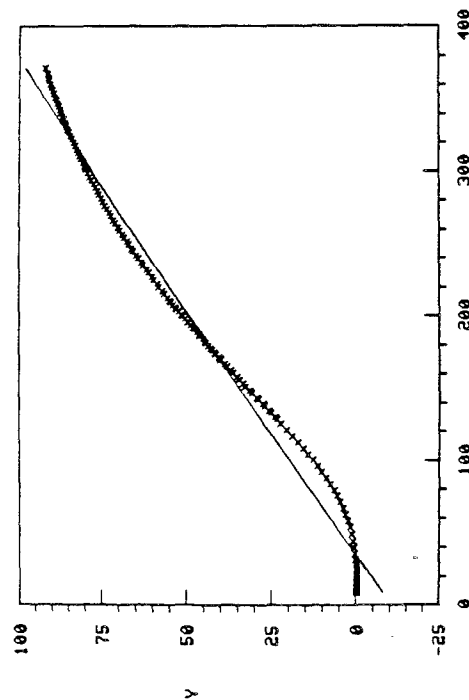
| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION X | PARAMETER ESTIMATES |
|---------------------|------------------------|-------------------------------------|--------------------------------|
| 1-- | .10000-01 | .55094+01 X | -.23937+01 .27869+00 .25954-03 |
| 2-- | .50000-02 | .36329+01 X | -.10120+02 .29728+00 .45735-04 |
| 3-- | .25000-02 | .36250+01 X | -.10422+02 .30329+00 .11801-03 |
| 4-- | .12500-02 | .36248+01 X | -.10331+02 .30203+00 .10565-03 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1 A0 -10.3477 (.9393)
 2 A1 .302264 (.1016-01)
 3 B1 .108003-03 (.9839-04)

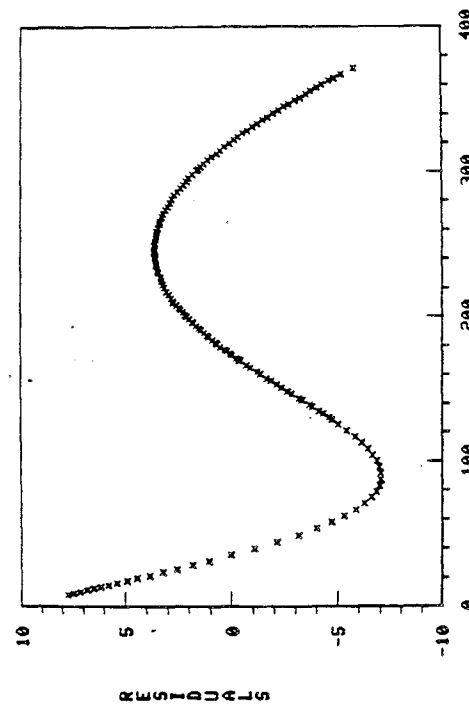
RESIDUAL STANDARD DEVIATION = 3.6248077750
 RESIDUAL DEGREES OF FREEDOM = 149
 REPLICATION STANDARD DEVIATION = .0120207869
 REPLICATION DEGREES OF FREEDOM = 1
 LACK OF FIT F RATIO = 91543.7461 - THE 99.7367% POINT OF THE
 F DISTRIBUTION WITH 148 AND 1 DEGREES OF FREEDOM

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



MODEL--Y = (A0 + A1X) / (1 + B1X)
 JJF6XCS9.NONLINEAR11 JJF6XDATA.KIRBY2 11/9/78

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



MODEL--Y = (A0 + A1X) / (1 + B1X)
 JJF6XCS9.NONLINEAR11 JJF6XDATA.KIRBY2 11/9/78

EXACT RATIONAL FUNCTION FIT
 NUMBER OF POINTS IN FIRST SET " 5
 DEGREE OF NUMERATOR " 2
 DEGREE OF DENOMINATOR " 2

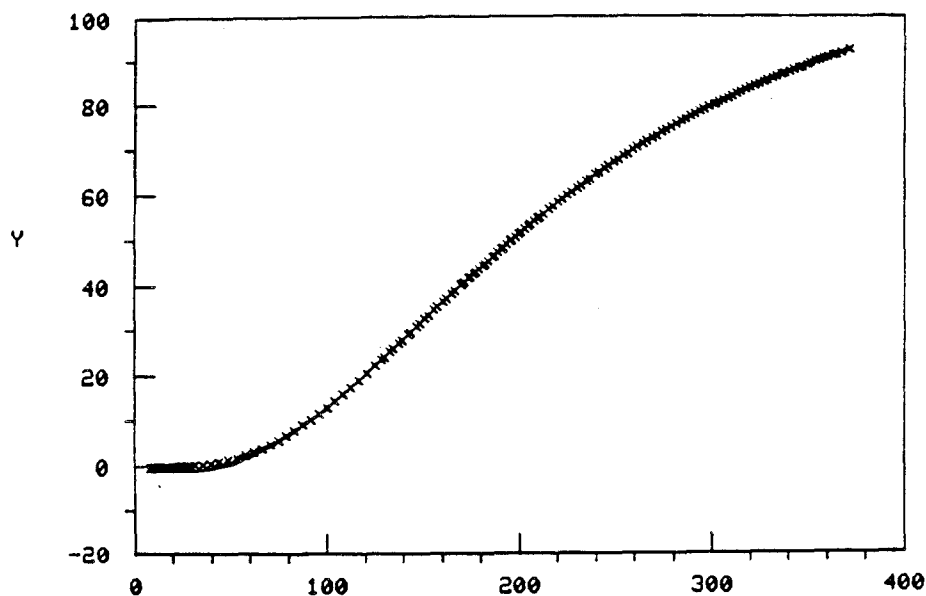
NUMERATOR --A0 A1 A2 " .125+01 -.170+00
 .304-02
 DENOMINATOR--B0 B1 B2 " .100+01 -.105-02
 .236-04

APPLICATION OF EXACT-FIT COEFFICIENTS
 TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET " 152
 NUMBER OF ESTIMATED COEFFICIENTS " 5
 RESIDUAL DEGREES OF FREEDOM " 147

RESIDUAL SUM OF SQUARES " .23364094+02
 RESIDUAL STANDARD DEVIATION (DENOM=N-P) " .39867206+00
 AVERAGE ABSOLUTE RESIDUAL (DENOM=N) " .17420260+00
 LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL " .14435167+01
 LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL " -.15705872+00
 LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL " .14435167+01

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS



EXACT FIT THROUGH 5 POINTS OF QUADRATIC/QUADRATIC MODEL
 JJF6*CS9.NONLINEAR11 JJF6*DATA.KIRBY2 11/9/78

LEAST SQUARES NON-LINEAR FIT

| | |
|----------------------------------|-----------------------------|
| SAMPLE SIZE N = | 152 |
| MODEL-- Y = | (HO+A1X+A2X^2)/1518X^28X^N) |
| REPLICATION CASE | |
| REPLICATION STANDARD DEVIATION = | .1202076887-01 |
| NUMBER OF DEGREES OF FREEDOM = | 1 |
| NUMBER OF DISTINCT SUBSETS = | 151 |

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES | | |
|---------------------|------------------------|-----------------------------------|------------------------|------------|------------|
| 1 | .10000-.01 | .39867+00 | .12459+01 | - | .10546-.02 |
| 2 | .50000-.02 | .18310+00 | .18670+00 | - | .14969-.02 |
| 3 | .25000-.02 | .17138+00 | .15963+01 | - | .17291-.02 |
| 4 | .12500-.02 | .17107+00 | .15629+01 | - | .17530-.02 |
| 5 | .62500-.03 | .17106+00 | .15635+01 | - | .17521-.02 |
| | | | | .30363-.02 | .23646-.04 |
| | | | | .27499-.02 | .22306-.04 |
| | | | | .25808-.02 | .21566-.04 |
| | | | | .25620-.02 | .21483-.04 |
| | | | | .25625-.02 | .21486-.04 |

FINAL PARAMETER ESTIMATES

| | | | |
|---|----|-------------|------------|
| 1 | A0 | 1.56345 | (.8621-01) |
| 2 | A1 | -.135275 | (.4119-02) |
| 3 | A2 | .256242-02 | (.4224-04) |
| 4 | B1 | -.175250-02 | (.5988-04) |
| 5 | B2 | .214859-04 | (.2052-06) |

RESIDUAL STANDARD DEVIATION = .1710639019

RESIDUAL DEGREES OF FREEDOM • 147

REPLICATION STANDARD DEVIATION * .0120207869

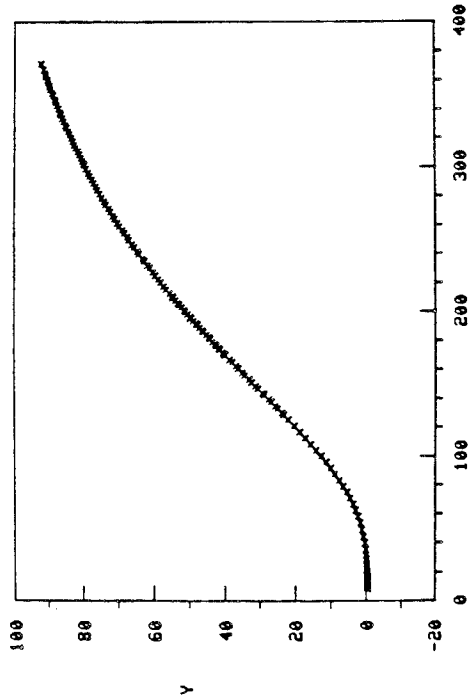
REPLICATION DEGREES OF FREEDOM - 1

REASON FOR REJECTION: LACK OF FIT F RATIO = 203.8923 - THE 94.4264% POINT OF THE

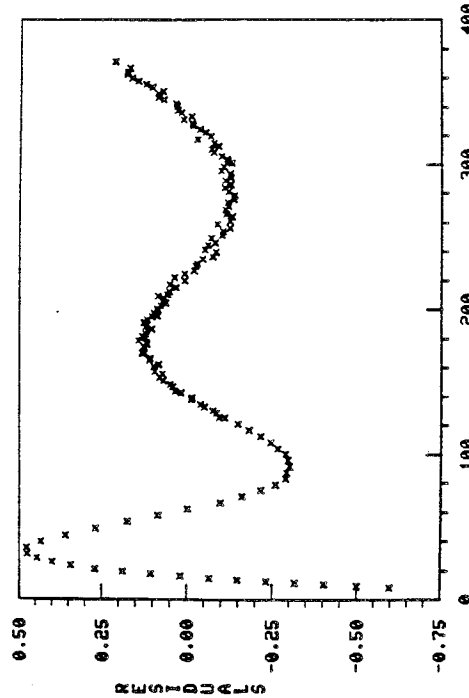
1 DEGREES OF FREEDOM

THE

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS


$$\text{MODEL--Y} = (A0 + A1X + A2X^2X) / (1 + B1X + B2X^2X)$$

SCANNING ELECTRON MICROSCOPE LINE SPACING STANDARDS


$$\text{MODEL} \rightarrow Y = (A0 + A1X + A2X^2) / (1 + B1X + B2X^2)$$

EXAMPLE 20

COMMENT EXAMPLE--BILL KEERY SEMICONDUCTOR DIFFUSION STUDY
 COMMENT MODEL --QUADRATIC/QUADRATIC
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
 COMMENT NOTE --A SECOND FIT RESTRICTED TO GO THROUGH ORIGIN

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6*DATA.KEERY1 Y X

CHARACTERS X
 LINES
 PLOT Y X

LET ID = 0*Y
 LET ID(1)=1
 LET ID(3)=1
 LET ID(5)=1
 LET ID(9)=1
 LET ID(19)=1
 LET Y2 = Y SUBSET ID 1
 LET X2 = X SUBSET ID 1
 PACK Y2 X2 SUBSET ID 1
 EXACT 2/2 FIT Y2 X2 Y X

TITLE SEMICONDUCTOR BORON DIFFUSION

YLABEL CONCENTRATION

XLABEL DEPTH

X2LABEL EXACT QUADRATIC/QUADRATIC RATIONAL FIT

X3LABEL JJF6*CS9.NONLINEAR28 JJF6*DATA.KEERY1

4/78

CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Y PRED US X
 LET S1=RESSD

FIT Y = (A0+A1XX+A2XXX)/(1+B1XX+B2XXX)

X2LABEL MODEL--Y = (A0 + A1X + A2XXX) / (1 + B1X + B2XXX)

PLOT Y PRED US X

LET S2=RESSD

LET MN=MIN(X)

LET MX=MAX(X)

LET DELTA=(MX-MN)/100

PLOT Y X AND

PLOT Y = (A0+A1XX+A2XXX)/(1+B1XX+B2XXX) FOR X = MN DELTA MX

FIT Y = (0+A1XX+A2XXX)/(1+B1XX+B2XXX)

X2LABEL MODEL--Y = (A1XX + A2XXX) / (1 + B1X + B2XXX)

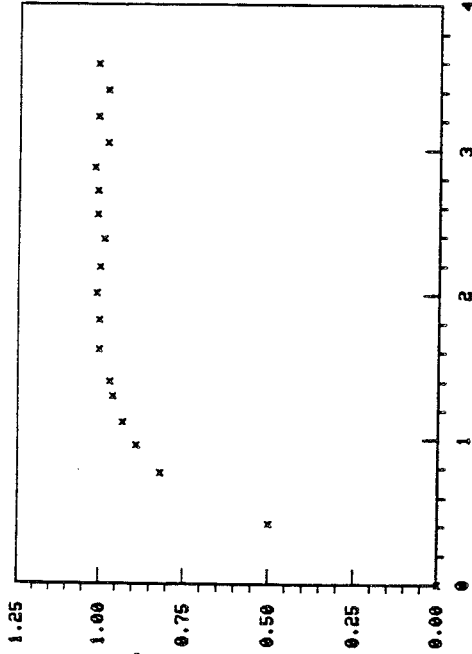
PLOT Y PRED US X

LET S3=RESSD

PLOT Y X AND

PLOT Y = (A1XX+A2XXX)/(1+B1XX+B2XXX) FOR X = MN DELTA MX

PRINT S1 S2 S3



EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 5 |
| DEGREE OF NUMERATOR | = | 2 |
| DEGREE OF DENOMINATOR | = | 2 |

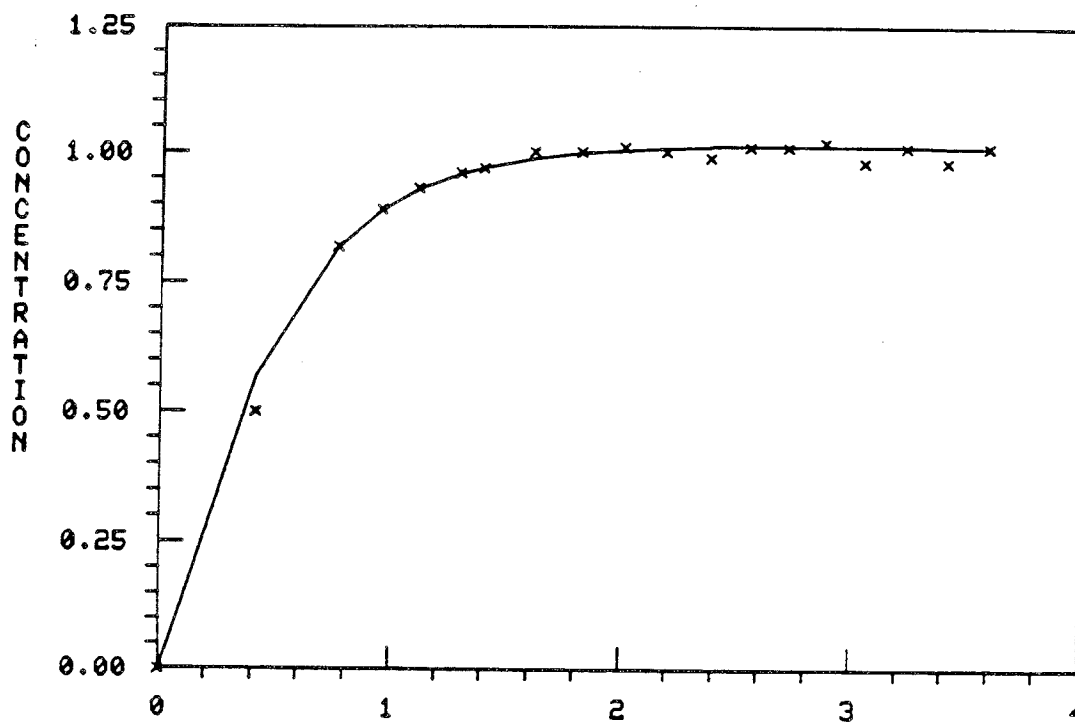
| | | | | | | |
|-------------|------|----|----|---|---------|---------|
| NUMERATOR | --A0 | A1 | A2 | = | .000 | .144+01 |
| | | | | | | .191+01 |
| DENOMINATOR | --B0 | B1 | B2 | = | .100+01 | .692+00 |
| | | | | | | .201+01 |

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|----|
| NUMBER OF POINTS IN SECOND SET | = | 19 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 5 |
| RESIDUAL DEGREES OF FREEDOM | = | 14 |

| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .77758524-02 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .23567308-01 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .10378105-01 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .10173738-01 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.70652820-01 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .70652820-01 |

SEMICONDUCTOR BORON DIFFUSION



EXACT QUADRATIC/QUADRATIC RATIONAL FIT
JJF6*CS9.NONLINEAR28 JJF6*DATA.KEERY1 4/78

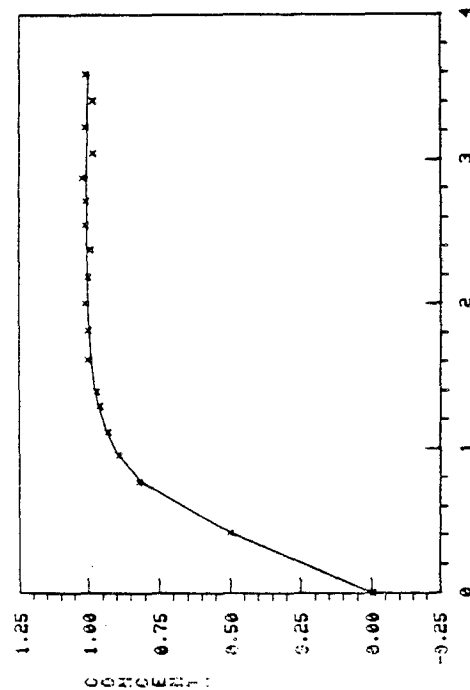
LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 19
MODEL--Y = (A0+A1X+A2XXX)/(1+B1X+B2XXX)
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES | | | |
|---------------------|------------------------|-----------------------------------|------------------------|-----------|-----------|------------|
| 1-- | .1000-01 | .23567-01 | .00000 | .14353+01 | .19057+01 | .69152+00 |
| 2-- | .5000-02 | .12872-01 | -.80934-03 | .62511+00 | .21116+01 | -.26193+00 |
| 3-- | .2500-02 | .11996-01 | -.23370-03 | .57693+00 | .24805+01 | -.26810+00 |
| 4-- | .1250-02 | .11901-01 | -.82112-04 | .49801+00 | .27022+01 | -.32683+00 |
| 5-- | .6250-03 | .11875-01 | .54263-03 | .47626+00 | .27544+01 | -.34253+00 |
| 6-- | .31250-03 | .11872-01 | -.91546-04 | .47673+00 | .27597+01 | -.34203+00 |
| | | | | | | .20124+01 |
| | | | | | | .22953+01 |
| | | | | | | .26497+01 |
| | | | | | | .28619+01 |
| | | | | | | .29101+01 |
| | | | | | | .29148+01 |

FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)
1 A0 (-.797385-04)
2 A1 (-.1187-01)
3 A2 (.3359)
4 B1 (.8703)
5 B2 (.2764)
.8478

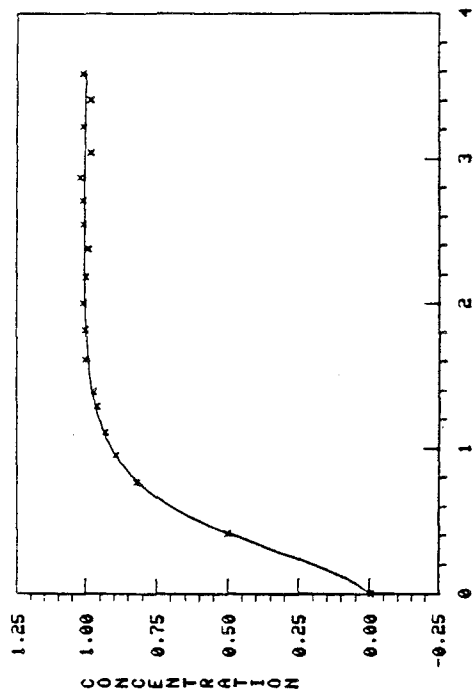
RESIDUAL STANDARD DEVIATION : .0118720974
DEGREES OF FREEDOM : 14

SEMICONDUCTOR BORON DIFFUSION



MODEL--Y = (A0 + A1X + A2XXX) / (1 + B1X + B2XXX)
JJF61CS9.NONLINEAR28 JJF61DATA.KEERV1 4/78

SEMICONDUCTOR BORON DIFFUSION



MODEL--Y = (A0 + A1X + A2XXX) / (1 + B1X + B2XXX)
JJF61CS9.NONLINEAR28 JJF61DATA.KEERV1 4/78

LEAST SQUARES NON-LINEAR FIT

SAMPLE SIZE N = 19
 MODEL--Y = (A1XX+A2XX)/(1+B1XX+B2XX)
 NO REPLICATION CASE

ITERATION CONVERGENCE RESIDUAL & PARAMETER
 NUMBER MEASURE STANDARD DEVIATION & ESTIMATES

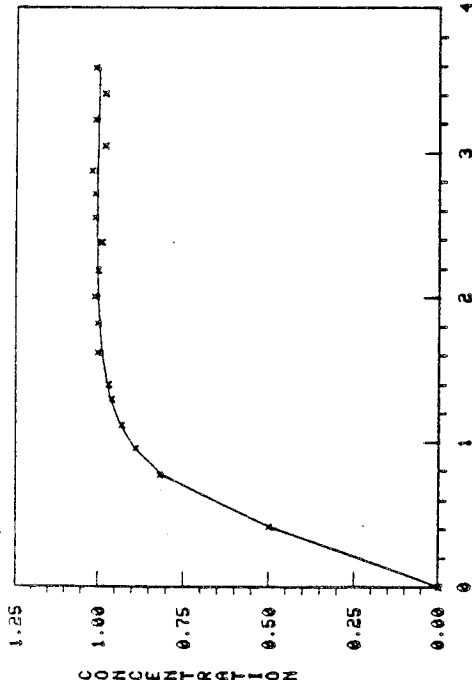
1-- .10000-01 .11470-01 .47674+00 .27598+01 -.34204+00 .29148+01

FINAL PARAMETER ESTIMATES (APPROX. ST. DEU.)

1 A1 .476347
 2 A2 2.75979
 3 B1 -.342327
 4 B2 2.91482

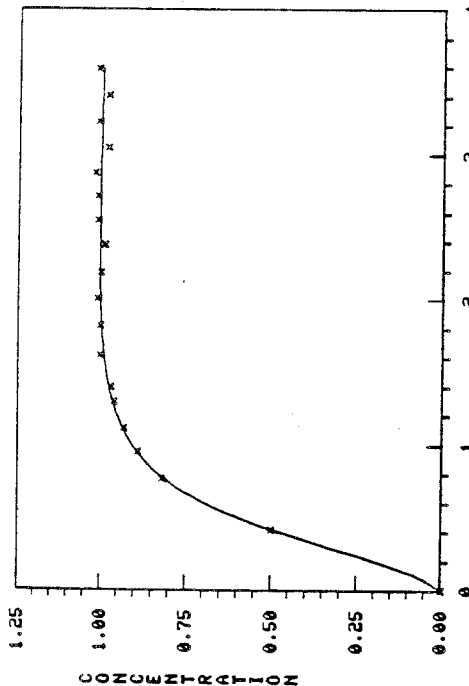
RESIDUAL STANDARD DEVIATION = .0114695737
 RESIDUAL DEGREES OF FREEDOM = 15

SEMICONDUCTOR BORON DIFFUSION



MODEL--Y = (A1XX + A2XX) / (1 + B1XX + B2XX)
 JJF64CS9.NONLINEAR28 JJF64DATA.KEERV1 4/78

SEMICONDUCTOR BORON DIFFUSION



MODEL--Y = (A1XX + A2XX) / (1 + B1XX + B2XX)
 JJF64CS9.NONLINEAR28 JJF64DATA.KEERV1 4/78

PARAMETERS AND CONSTANTS--

S1 -- .2356731-01
 S2 -- .1187210-01
 S3 -- .1146957-01

EXAMPLE 21

COMMENT EXAMPLE--TOM HAHN COPPER THERMAL EXPANSION STUDY
 COMMENT MODEL --QUADRATIC/QUADRATIC AND CUBIC/CUBIC
 COMMENT NOTE --TECHNIQUE FOR UPDATING AND IMPROVING
 COMMENT RATIONAL FIT MODEL
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
 COMMENT NOTE --A SECOND FIT RESTRICTED TO GO THROUGH ORIGIN

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF61DATA.HAHN1 X Y

CHARACTERS X
 LINES
 PLOT Y X

LET X2(1)=10
 LET X2(2)=50
 LET X2(3)=120
 LET X2(4)=200
 LET X2(5)=800
 LET Y2(1)=0
 LET Y2(2)=5
 LET Y2(3)=12
 LET Y2(4)=15
 LET Y2(5)=20
 EXACT 2/2 FIT Y2 X2 Y X

TITLE COEFFICIENT OF THERMAL EXPANSION (COPPER)
 YLABEL COEF. OF THERMAL EXP.
 XLABEL TEMPERATURE (DEGREES KELVIN)
 X2LABEL EXACT QUADRATIC/QUADRATIC RATIONAL FIT
 X3LABEL JJF61CS9.NONLINEAR44 JJF61DATA.HAHN1 11/17/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Y PRED US X
 CHARACTERS BLANK ALL
 LINES DOTTED SOLID
 PLOT Y PRED US X

LET X2(6)=40
 LET X2(7)=30
 LET Y2(6)=3
 LET Y2(7)=2
 EXACT 3/3 RATIONAL FIT Y2 X2 Y X

X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
 PLOT Y PRED US X

FIT Y = $(A0+A1X+A2X^2+A3X^3)/(1+B1X+B2X^2+B3X^3)$
 LET S1=RESSD

X2LABEL MODEL--Y = $(A0+A1X+A2X^2+A3X^3)/(1+B1X+B2X^2+B3X^3)$
 PLOT Y PRED US X

YLABEL RESIDUALS
 PLOT RES X

XLABEL
 NORMAL PROBABILITY PLOT RES

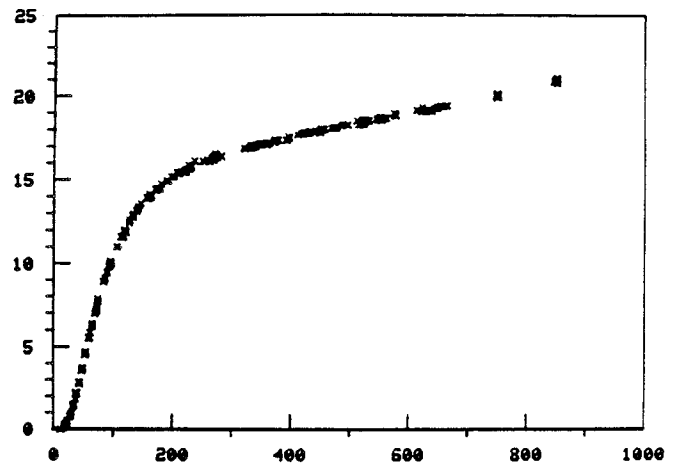
FIT Y = $(A1X+A2X^2+A3X^3)/(1+B1X+B2X^2+B3X^3)$
 LET S2=RESSD

YLABEL COEF. OF THERMAL EXP.
 XLABEL TEMPERATURE (DEGREES KELVIN)
 X2LABEL MODEL--Y = $(A1X+A2X^2+A3X^3)/(1+B1X+B2X^2+B3X^3)$
 PLOT Y PRED US X

YLABEL RESIDUALS
 PLOT RES X

XLABEL
 NORMAL PROBABILITY PLOT RES

PRINT S1 S2

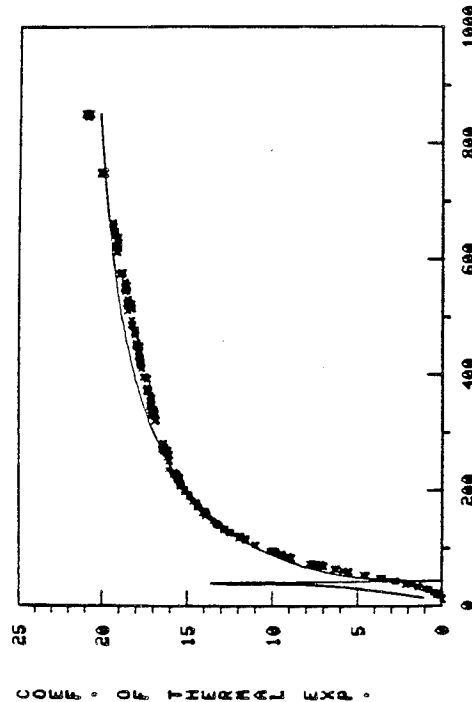


| | | | |
|-------------------------------|---|--|----------|
| EXACT RATIONAL FUNCTION FIT | | | |
| NUMBER OF POINTS IN FIRST SET | - | | 5 |
| DEGREE OF NUMERATOR | - | | 2 |
| DEGREE OF DENOMINATOR | - | | 2 |
| | | | |
| NUMERATOR --A0 A1 A2 | - | | |
| -.683-02 | | | .369+00 |
| DENOMINATOR--B0 B1 B2 | - | | |
| -.306-03 | | | -.112-01 |

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

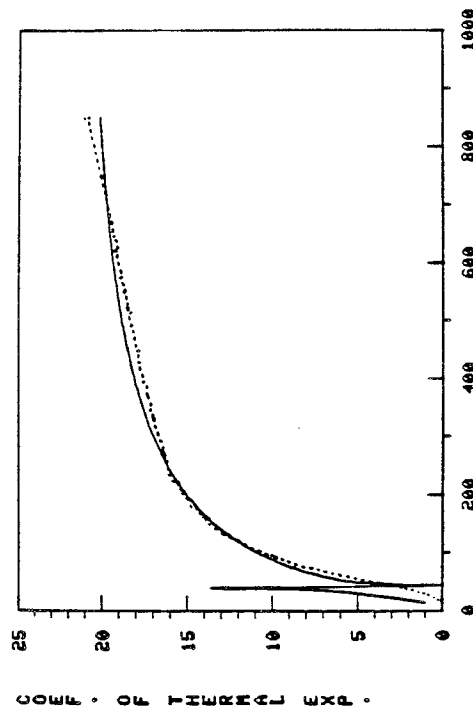
| | | |
|--|---|---------------|
| NUMBER OF POINTS IN SECOND SET | - | 236 |
| NUMBER OF ESTIMATED COEFFICIENTS | - | 5 |
| RESIDUAL DEGREES OF FREEDOM | - | 231 |
| | | |
| RESIDUAL SUM OF SQUARES | - | .68716166+03 |
| RESIDUAL STANDARD DEVIATION (DENOM-N-P) | - | .17247393+01 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM-N) | - | .82942525+00 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | - | .27054440+01 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | - | -.11427686+02 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | - | .11427686+02 |

COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
EXACT QUADRATIC/QUADRATIC RATIONAL FIT
JUF63CS9.NONLINEAR44 JUF63DATA.HANH1 11/17/78

COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
EXACT QUADRATIC/QUADRATIC RATIONAL FIT
JUF63CS9.NONLINEAR44 JUF63DATA.HANH1 11/17/78

EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 7 |
| DEGREE OF NUMERATOR | = | 3 |
| DEGREE OF DENOMINATOR | = | 3 |

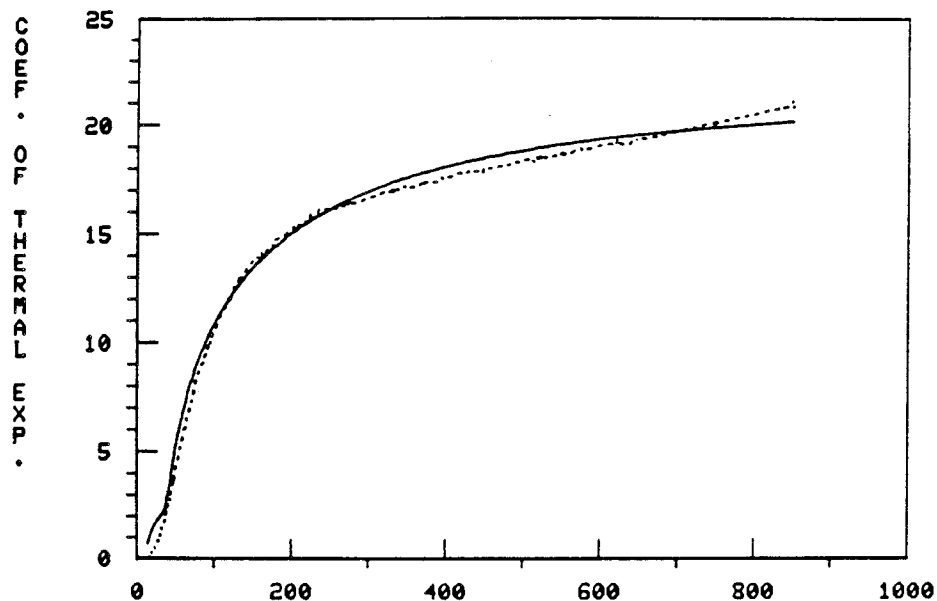
| | | | | | | | | |
|-------------|------|--------------|----|----|---|---------------|---------------|----|
| NUMERATOR | --A0 | A1 | A2 | A3 | = | -.23235123+01 | .35300880+00 | -. |
| 13832568-01 | | .17668116-03 | | | | | | |
| DENOMINATOR | --B0 | B1 | B2 | B3 | = | .10000000+01 | -.33957982-01 | . |
| 11009136-03 | | .79103667-05 | | | | | | |

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|-----|
| NUMBER OF POINTS IN SECOND SET | = | 236 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 7 |
| RESIDUAL DEGREES OF FREEDOM | = | 229 |

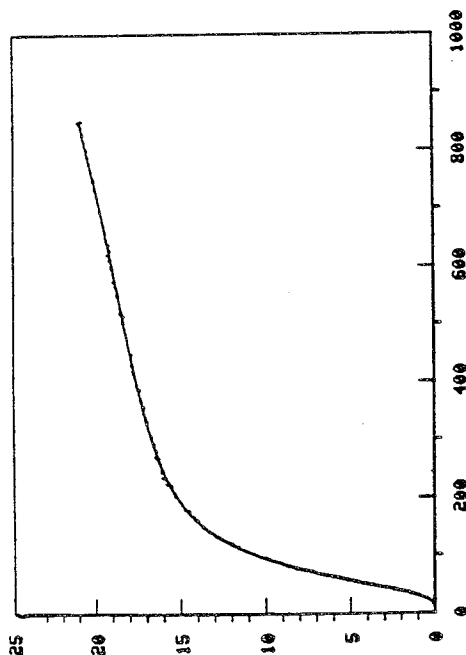
| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .78287830+02 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .58469500+00 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .47006864+00 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .95732951+00 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.13508249+01 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .13508249+01 |

COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
EXACT CUBIC/CUBIC RATIONAL FIT
JJF6*CS9.NONLINEAR44 JJF6*DATA.HAHN1 11/17/78

U.S. GOVERNMENT PRINTING OFFICE



```

TEMPERATURE (DEGREES KELVIN)
MODEL--Y = (A0+A1X+A2X**X+A3X**X**3)/(1+B1X+B2X**X+B3X**X**3)
JJFG8CS9,NONLINEAR44 JJFG8DATA.HANH1 11/17/78

```

[illegible]

| | |
|--------------------------------|--|
| MODEL ID# N | 236 |
| MODEL - Y | (A0-010K020200-0-300023)/1-0100-0000XIX-02300033 |
| REPLICATION CASE | |
| REPLICATION STANDARD DEVIATION | |
| REPLICATION DEGREE OF FREEDOM | .0131726601-01 |
| NUMBER OF DISTINCT SUBSTRATE | 236 |

| REGRESSION NUMBER | CONFIDENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|----------------------|-----------------------|-----------------------------------|------------------------|
|----------------------|-----------------------|-----------------------------------|------------------------|

[illegible]

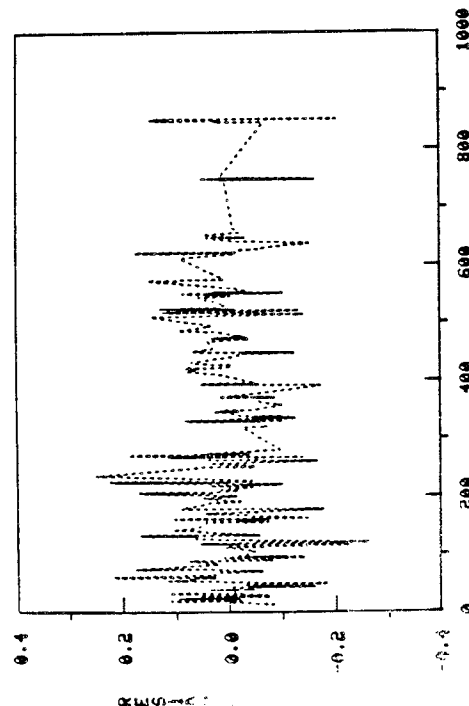
FINAL PENDING ESTIMATES

| | FINAL PARAMETER ESTIMATES | (APPROX. ST. DEV.) |
|-----|---------------------------|--------------------|
| 1 | .40 | (.1078) |
| 2 | 1.00 | (.1195) |
| 3 | 1.00 | (.1256) |
| 4 | 1.00 | (.1270) |
| 5 | .42 | (.1421) |
| 6 | .42 | (.1421) |
| 7 | .42 | (.1421) |
| 8 | .42 | (.1421) |
| 9 | .42 | (.1421) |
| 10 | .42 | (.1421) |
| 11 | .42 | (.1421) |
| 12 | .42 | (.1421) |
| 13 | .42 | (.1421) |
| 14 | .42 | (.1421) |
| 15 | .42 | (.1421) |
| 16 | .42 | (.1421) |
| 17 | .42 | (.1421) |
| 18 | .42 | (.1421) |
| 19 | .42 | (.1421) |
| 20 | .42 | (.1421) |
| 21 | .42 | (.1421) |
| 22 | .42 | (.1421) |
| 23 | .42 | (.1421) |
| 24 | .42 | (.1421) |
| 25 | .42 | (.1421) |
| 26 | .42 | (.1421) |
| 27 | .42 | (.1421) |
| 28 | .42 | (.1421) |
| 29 | .42 | (.1421) |
| 30 | .42 | (.1421) |
| 31 | .42 | (.1421) |
| 32 | .42 | (.1421) |
| 33 | .42 | (.1421) |
| 34 | .42 | (.1421) |
| 35 | .42 | (.1421) |
| 36 | .42 | (.1421) |
| 37 | .42 | (.1421) |
| 38 | .42 | (.1421) |
| 39 | .42 | (.1421) |
| 40 | .42 | (.1421) |
| 41 | .42 | (.1421) |
| 42 | .42 | (.1421) |
| 43 | .42 | (.1421) |
| 44 | .42 | (.1421) |
| 45 | .42 | (.1421) |
| 46 | .42 | (.1421) |
| 47 | .42 | (.1421) |
| 48 | .42 | (.1421) |
| 49 | .42 | (.1421) |
| 50 | .42 | (.1421) |
| 51 | .42 | (.1421) |
| 52 | .42 | (.1421) |
| 53 | .42 | (.1421) |
| 54 | .42 | (.1421) |
| 55 | .42 | (.1421) |
| 56 | .42 | (.1421) |
| 57 | .42 | (.1421) |
| 58 | .42 | (.1421) |
| 59 | .42 | (.1421) |
| 60 | .42 | (.1421) |
| 61 | .42 | (.1421) |
| 62 | .42 | (.1421) |
| 63 | .42 | (.1421) |
| 64 | .42 | (.1421) |
| 65 | .42 | (.1421) |
| 66 | .42 | (.1421) |
| 67 | .42 | (.1421) |
| 68 | .42 | (.1421) |
| 69 | .42 | (.1421) |
| 70 | .42 | (.1421) |
| 71 | .42 | (.1421) |
| 72 | .42 | (.1421) |
| 73 | .42 | (.1421) |
| 74 | .42 | (.1421) |
| 75 | .42 | (.1421) |
| 76 | .42 | (.1421) |
| 77 | .42 | (.1421) |
| 78 | .42 | (.1421) |
| 79 | .42 | (.1421) |
| 80 | .42 | (.1421) |
| 81 | .42 | (.1421) |
| 82 | .42 | (.1421) |
| 83 | .42 | (.1421) |
| 84 | .42 | (.1421) |
| 85 | .42 | (.1421) |
| 86 | .42 | (.1421) |
| 87 | .42 | (.1421) |
| 88 | .42 | (.1421) |
| 89 | .42 | (.1421) |
| 90 | .42 | (.1421) |
| 91 | .42 | (.1421) |
| 92 | .42 | (.1421) |
| 93 | .42 | (.1421) |
| 94 | .42 | (.1421) |
| 95 | .42 | (.1421) |
| 96 | .42 | (.1421) |
| 97 | .42 | (.1421) |
| 98 | .42 | (.1421) |
| 99 | .42 | (.1421) |
| 100 | .42 | (.1421) |

PE SIGNAL STANDARD DEVIATION • .018038499

| | | | |
|--------------------|--------------------|---------|------------------------|
| RESIDUAL | STANDARD DEVIATION | • | 0.018235490 |
| DEGREES OF FREEDOM | • | 249 | |
| RESIDUAL | STANDARD DEVIATION | • | 0.013172540 |
| DEGREES OF FREEDOM | • | 249 | |
| LIKELIHOOD RATIO | • | THE | 32.126549 POINT OF THE |
| TEST | • | 1.0181 | • |
| WILCOXSON | • | 249 AND | 1 MODELS OF FREEDOM |
| TEST | • | 249 AND | 1 MODELS OF FREEDOM |

COEFFICIENT OF THERMAL EXPANSION (COPPER)



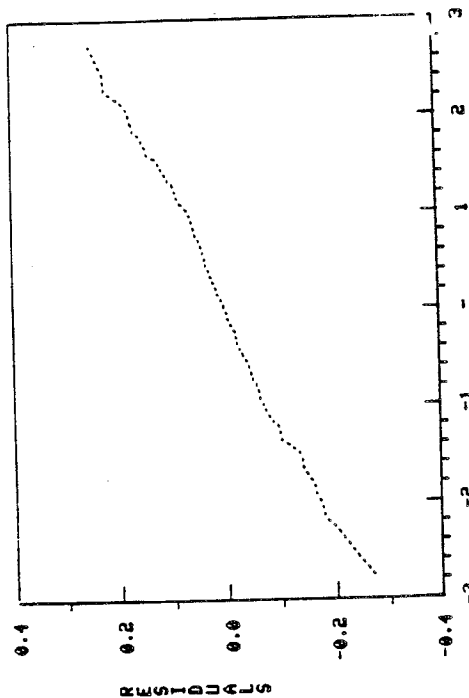
```

TEMPERATURE (DEGREES KELVIN)
MODEL--Y = (A0+A1X+A2XX+A3XXX+3)/(1+B1X+B2XX+B3XXX)
JJF61C59,NONLINEAR44 JJF61DATA.HANN1 11/17/78

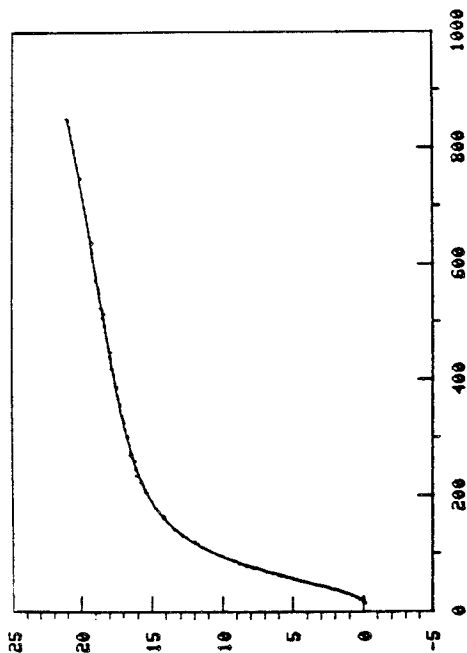
```

MODEL--Y - (A0+A1X+A2X^2+A3X^3)/(1+B1X+B2X^2+B3X^3)
JJF6XCS9.NONLINEAR44 JJF6XDATA.HAHNI 11/17/78

COEFFICIENT OF THERMAL EXPANSION (COPPER)

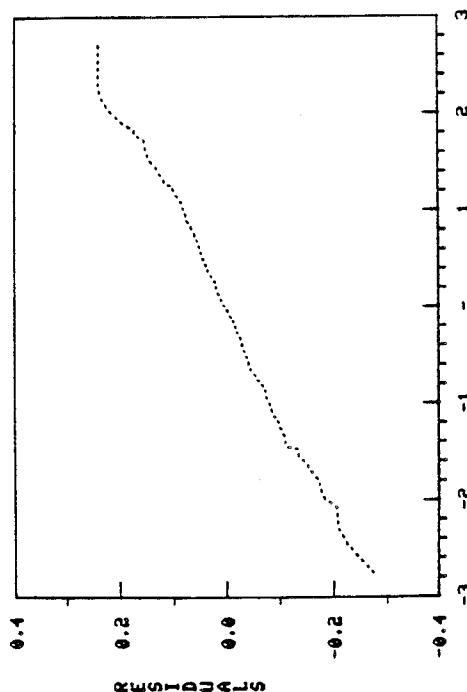


COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
 MODEL--Y = (A1XX+A2XX+A3XX)/((1+B1XX+B2XX+B3XX)³)
 JJF6SCS9.NONLINEAR44 JJF6DATA.HAHN1 11/17/78

COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
 MODEL--Y = (A1XX+A2XX+A3XX)/((1+B1XX+B2XX+B3XX)³)
 JJF6SCS9.NONLINEAR44 JJF6DATA.HAHN1 11/17/78

C O E F F I C I E N T O F T H E R M A L E X P A N S I O N

10000-01 -11429-00 8 -12277-00 -6878-02 -14279-05 -57690-02
 20000-02 -11661-00 8 -12578-01 -3552-02 -13763-05 -32625-02
 30000-03 -9428-01 8 -8293-01 -20511-03 -81784-06 -58929-05
 40000-04 -8034-01 8 -51601-01 -28504-02 -31131-06 -51577-02
 50000-05 -89313-01 8 -51602-01 -28848-02 -39408-06 -51006-02
 60000-06 -89313-01 8 -51602-01 -28848-02 -39408-06 -51006-02

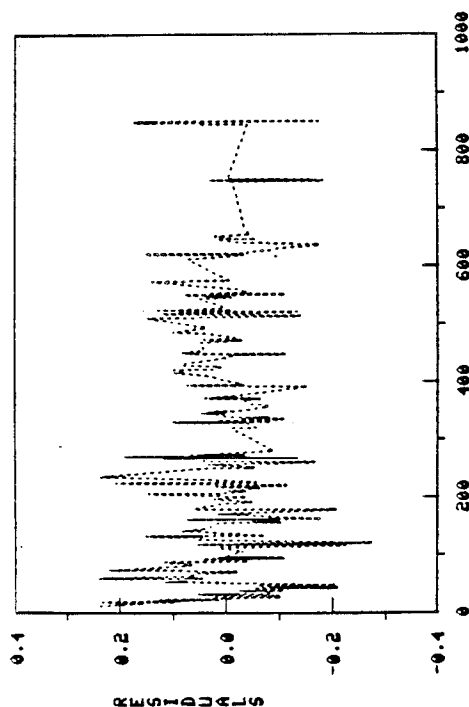
FINAL PARAMETER ESTIMATES (APPROX. ST. DEV.)

1 A1 -51429-01 (.827-02)
 2 A2 -288430-02 (.0400-04)
 3 A3 -20260-08 (.1916-05)
 4 B1 -518100-02 (.263-02)
 5 B2 -18200-02 (.1820-02)
 6 B3 -640213-07 (.271-02)
 RESIDUAL STANDARD DEVIATION = .002128153
 DEGREES OF FREEDOM = 230
 REPLICATION STANDARD DEVIATION = .0013172660
 DEGREES OF FREEDOM = 1
 LACK OF FIT F RATIO = 1.2045 • THE 36.31CRN POINT OF THE
 F DISTRIBUTION WITH 229 AND 1 DEGREES OF FREEDOM

LEAST SQUARES NON-LINEAR FIT
 SAMPLE SIZE N = 236
 MODEL--Y = (A1XX+A2XX+A3XX)/((1+B1XX+B2XX+B3XX)³)
 REPLICATION CASE
 REPLICATION STANDARD DEVIATION = .0131726601-01
 REPLICATION DEGREES OF FREEDOM = 1
 NUMBER OF DISTINCT SUBSETS = 236

REGRESSION MEASURE RESIDUAL STANDARD DEVIATION
 DEVIATION

COEFFICIENT OF THERMAL EXPANSION (COPPER)



TEMPERATURE (DEGREES KELVIN)
 MODEL--Y = (A1XX+A2XX+A3XX)/((1+B1XX+B2XX+B3XX)³)
 JJF6SCS9.NONLINEAR44 JJF6DATA.HAHN1 11/17/78

PARAMETERS AND CONSTANTS---

S1 -- .8180385-01
 S2 -- .8921282-01

EXAMPLE 22

COMMENT EXAMPLE--WARREN HAYES SMOKE OBSCURATION STUDY
 COMMENT MODEL --LINEAR/QUADRATIC AND LINEAR/QUARTIC
 COMMENT NOTE --TECHNIQUE FOR UPDATING AND IMPROVING
 COMMENT RATIONAL FIT MODEL
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF61DATA.HAYES3 X ST Y

CHARACTERS X
 LINES
 PLOT Y X

LET X2(1)=0
 LET X2(2)=100
 LET X2(3)=400
 LET X2(4)=800
 LET Y2(1)=0
 LET Y2(2)=2
 LET Y2(3)=.5
 LET Y2(4)=.1

EXACT 1/2 RATIONAL FIT Y2 X2 Y X

TITLE SMOKE OBSCURATION STUDY

YLABEL OBSCURATION

XLABEL TIME

X2LABEL EXACT LINEAR/QUADRATIC RATIONAL FIT

X3LABEL JJF61CS9.NONLINEAR36 JJF61DATA.HAYES3

CHARACTERS X BLANK

LINES BLANK SOLID

PLOT Y PRED US X

LET X2(5)=110
 LET Y2(5)=2
 LET X2(6)=270
 LET Y2(6)=1

EXACT 1/4 RATIONAL FIT Y2 X2 Y X

X2LABEL EXACT LINEAR/QUARTIC RATIONAL FIT

PLOT Y PRED US X

FIT Y = (A0+A1X)/(1+B1X+B2X**2+B3X**3+B4X**4)

X2LABEL MODEL--Y = (A0+A1X)/(1+B1X+B2X**2+B3X**3+B4X**4)

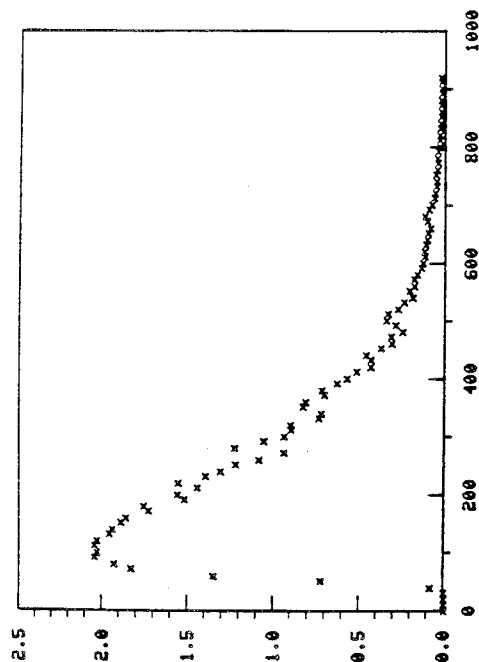
PLOT Y PRED US X

YLABEL RESIDUALS

PLOT RES X

XLABEL

NORMAL PROBABILITY PLOT RES



EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 4 |
| DEGREE OF NUMERATOR | = | 1 |
| DEGREE OF DENOMINATOR | = | 2 |

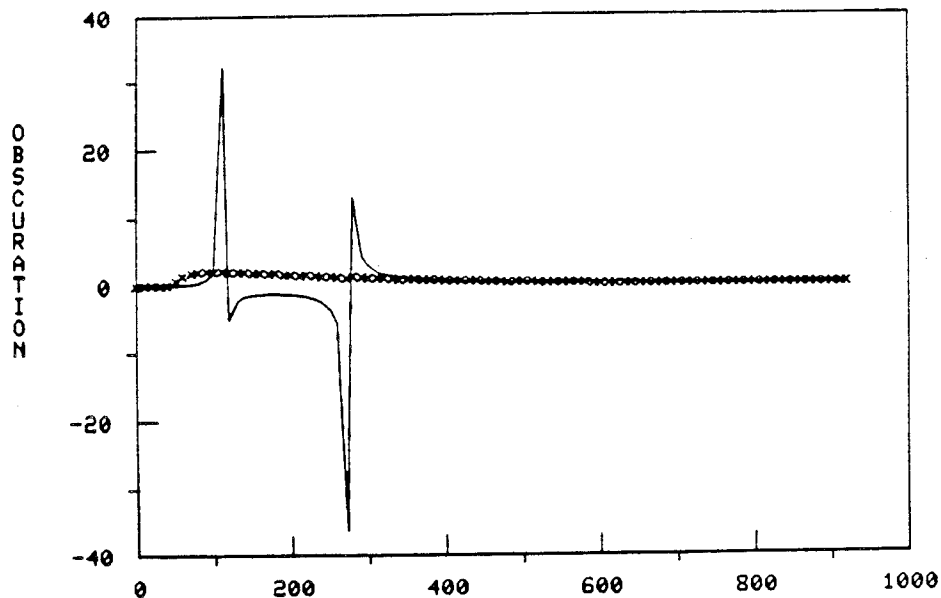
| | | | | | | |
|-------------|------|----|----|-----------|--------------|----------|
| NUMERATOR | --A0 | A1 | = | .00000000 | .14583332-02 | |
| DENOMINATOR | --B0 | B1 | B2 | = | .100+01 | -.125-01 |
| | | | | | .323-04 | |

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|----|
| NUMBER OF POINTS IN SECOND SET | = | 93 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 4 |
| RESIDUAL DEGREES OF FREEDOM | = | 89 |

| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .27069581+04 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .55150031+01 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .16479395+01 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .37217266+02 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.30199815+02 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .37217266+02 |

SMOKE OBSCURATION STUDY



EXACT LINEAR/QUADRATIC RATIONAL FIT
JJF6*CS9.NONLINEAR36 JJF6*DATA.HAYES3 11/18/78

EXACT RATIONAL FUNCTION FIT

NUMBER OF POINTS IN FIRST SET = 6
 DEGREE OF NUMERATOR = 1
 DEGREE OF DENOMINATOR = 4

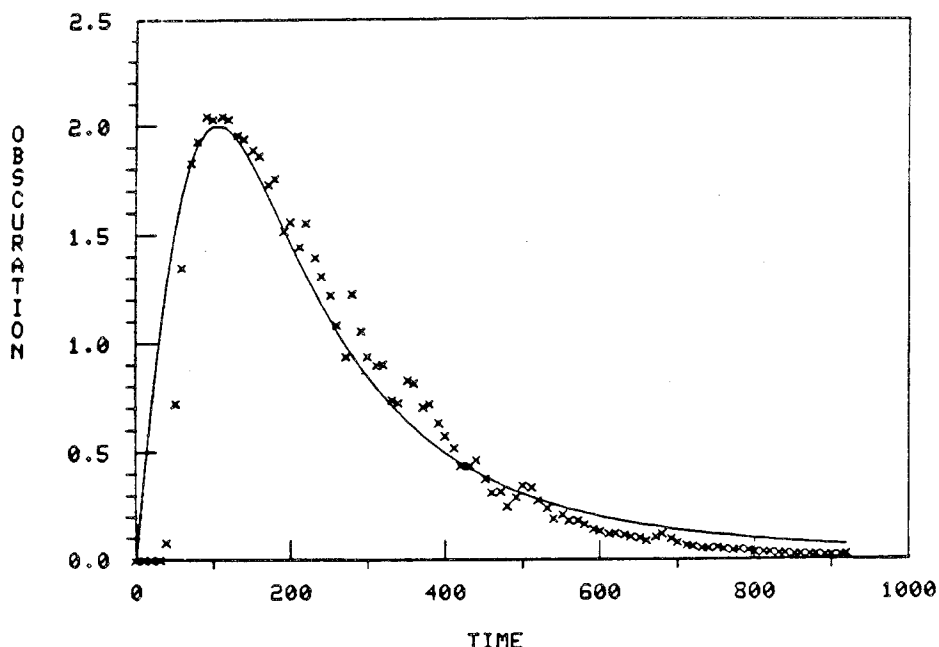
NUMERATOR --A0 A1 * .00000000 .37565201-01
 DENOMINATOR--B0 B1 B2 B3 B4 * .10000000+01 .20631021-02 .51
 269280-04 .10805190-06 .51205132-09

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

NUMBER OF POINTS IN SECOND SET = 93
 NUMBER OF ESTIMATED COEFFICIENTS = 6
 RESIDUAL DEGREES OF FREEDOM = 87

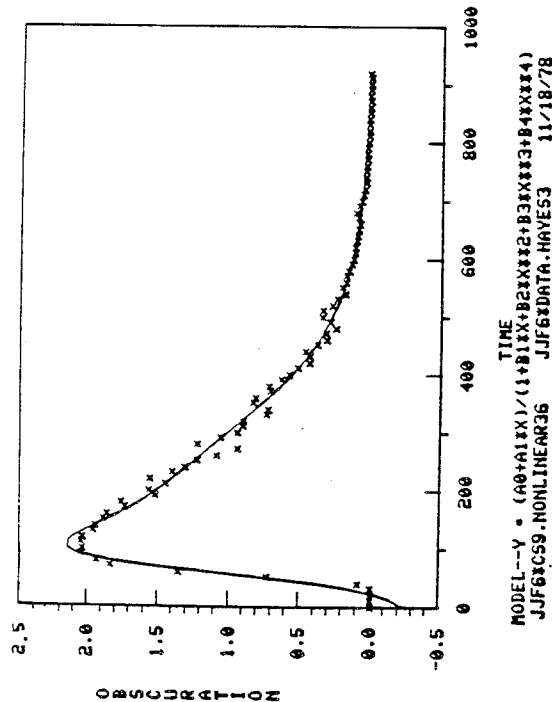
RESIDUAL SUM OF SQUARES = .46780170+01
 RESIDUAL STANDARD DEVIATION (DENOM=N-P) = .23188426+00
 AVERAGE ABSOLUTE RESIDUAL (DENOM=N) = .11407338+00
 LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL = .27681495+00
 LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL = -.11952349+01
 LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL = .11952349+01

SMOKE OBSCURATION STUDY



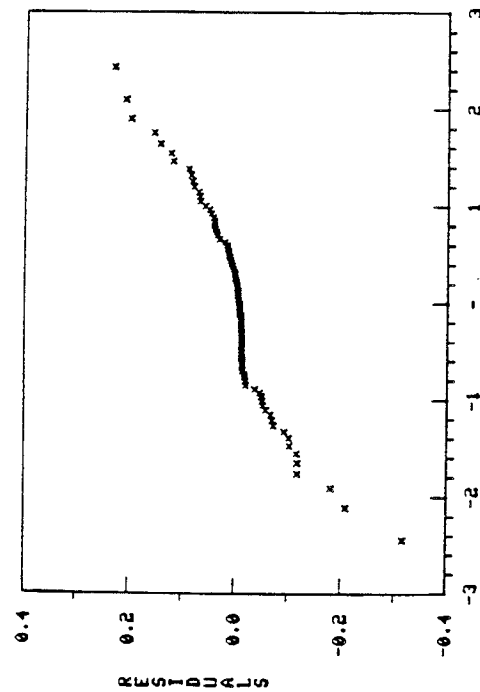
EXACT LINEAR/QUARTIC RATIONAL FIT
 JJF6*CS9.NONLINEAR36 JJF6*DATA.HAYES3 11/18/78

SMOKE OBSCURATION STUDY



MODEL--Y = (A0+A1X)/(1+B1X+B2X**2+B3X**3+B4X**4)
JUF6*CS9.NONLINEAR36 JUF6*DATA.HAYES3 11/18/78

SMOKE OBSCURATION STUDY



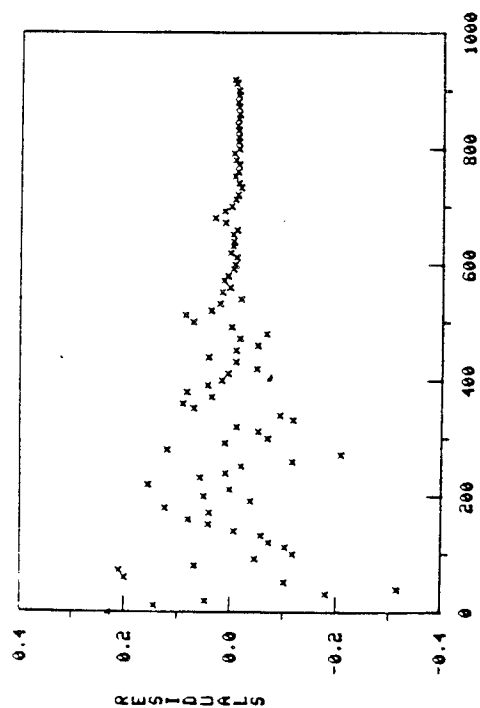
MODEL--Y = (A0+A1X)/(1+B1X+B2X**2+B3X**3+B4X**4)
JUF6*CS9.NONLINEAR36 JUF6*DATA.HAYES3 11/18/78

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 93
NUMBER OF PARAMETERS = 5
NO REPLICATION CASE

| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|------------------|---------------------|-----------------------------|---------------------|
| 1 | .1000E+01 | .23188E+00 | .37565E-01 |
| 2 | .75937E-01 | .23228E+00 | .17405E-01 |
| 3 | .77669E-01 | .13974E+00 | .51664E-01 |
| 4 | .48823E-01 | .13477E+00 | .37565E-01 |
| 5 | .47461E-01 | .86335E-01 | .17405E-01 |
| 6 | .23130E-02 | .80729E-01 | .51664E-01 |
| 7 | .11085E-02 | .79679E-01 | .37565E-01 |
| 8 | .52325E-03 | .78515E-01 | .17405E-01 |
| 9 | .24832E-03 | .78530E-01 | .51664E-01 |
| 10 | | | |
| 11 | | | |

FINAL PARAMETER ESTIMATES (IMPROX. ST. DEV.)
1 A0 (-2.30448) (.4279E-01)
2 A1 (-1.06142E-01) (.1132E-01)
3 B1 (-2.16763E-01) (.5529E-01)
4 B2 (-7.10692E-02) (.4887E-01)
5 B3 (-9.64631E-02) (.7576E-01)
6 B4
RESIDUAL STANDARD DEVIATION : .0795088410
DEGREES OF FREEDOM : 87

SMOKE OBSCURATION STUDY



MODEL--Y = (A0+A1X)/(1+B1X+B2X**2+B3X**3+B4X**4)
JUF6*CS9.NONLINEAR36 JUF6*DATA.HAYES3 11/18/78

EXAMPLE 23

COMMENT EXAMPLE--GEORGE MULHOLLAND DOPPLER SPECTROMETER EXAMPLE
 COMMENT MODEL --LINEAR/QUADRATIC
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT

ECHO ON
 HARDCOPY ON
 BELL ON

SKIP 1
 READ JJF6*DATA.MULHOLLAND2 X Y

CHARACTERS X
 LINES
 PLOT Y X

LET X2(1)=5.55
 LET X2(2)=5.65
 LET X2(3)=5.75
 LET X2(4)=5.85
 LET Y2(1)=150
 LET Y2(2)=400
 LET Y2(3)=600
 LET Y2(4)=200
 EXACT 1/2 RATIONAL FIT Y2 X2 Y X

TITLE PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)
 YLABEL DELTA N / DELTA D
 XLABEL DIAMETER
 X2LABEL EXACT LINEAR/QUADRATIC RATIONAL FIT
 X3LABEL JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Y PRED US X

FIT $Y = (A0 + A1X) / (1 + B1X + B2X^2)$

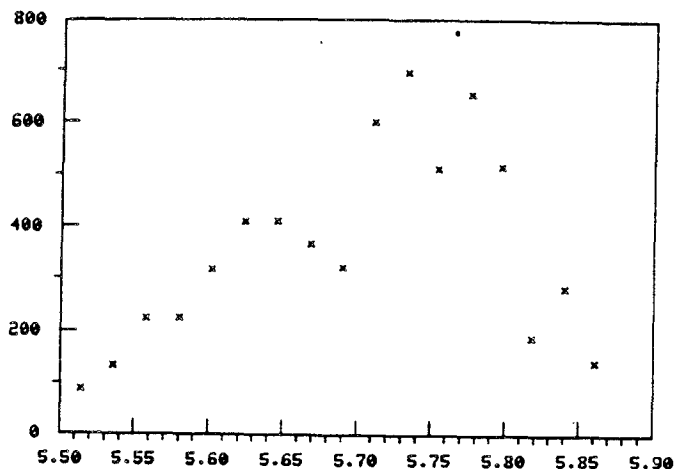
X2LABEL MODEL-- $Y = (A0 + A1X) / (1 + B1X + B2X^2)$
 PLOT Y PRED US X

LET MN=MIN(X)
 LET MX=MAX(X)
 LET DELTA=(MX-MN)/100

PLOT Y X AND
 PLOT $Y = (A0 + A1X) / (1 + B1X + B2X^2)$ FOR X = MN DELTA MX

YLABEL RESIDUALS
 PLOT RES X

XLABEL
 NORMAL PROBABILITY PLOT RES



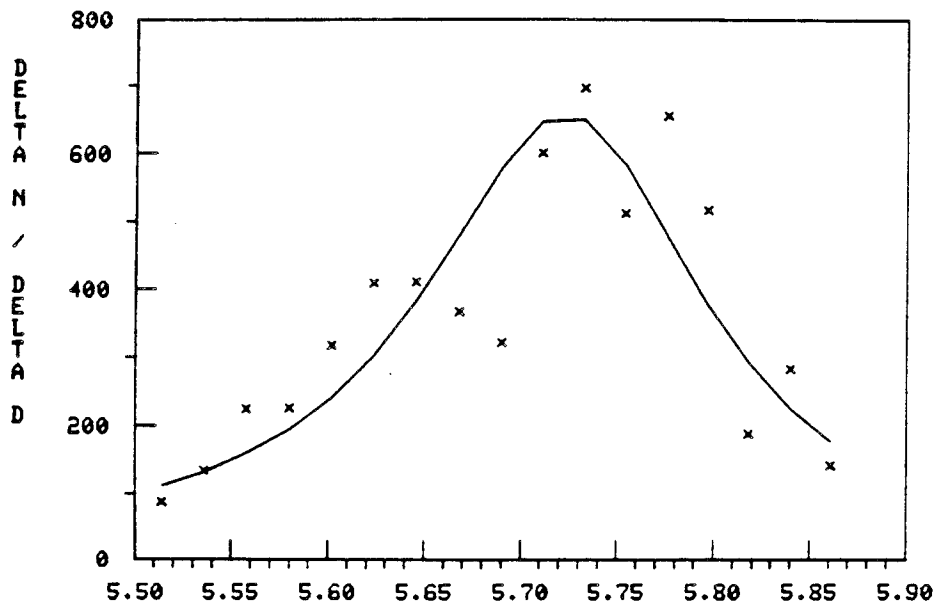
EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|----------------------------|
| NUMBER OF POINTS IN FIRST SET | = | 4 |
| DEGREE OF NUMERATOR | = | 1 |
| DEGREE OF DENOMINATOR | = | 2 |
| NUMERATOR --A0 A1 | = | .83234787+00 -.11806805+00 |
| DENOMINATOR--B0 B1 B2 | = | .100+01 -.349+00 |
| .305-01 | | |

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

| | | |
|--|---|---------------|
| NUMBER OF POINTS IN SECOND SET | = | 17 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 4 |
| RESIDUAL DEGREES OF FREEDOM | = | 13 |
| RESIDUAL SUM OF SQUARES | = | .17727561+06 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .11677579+03 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .80742574+02 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .17598337+03 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.25745164+03 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .25745164+03 |

PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



EXACT LINEAR/QUADRATIC RATIONAL FIT
 JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78

LEAST SQUARES NON-LINEAR FIT
SAMPLE SIZE N = 17
MODEL--Y = (A0+A1X)/(1+B1X+B2XX)
NO REPLICATION CASE

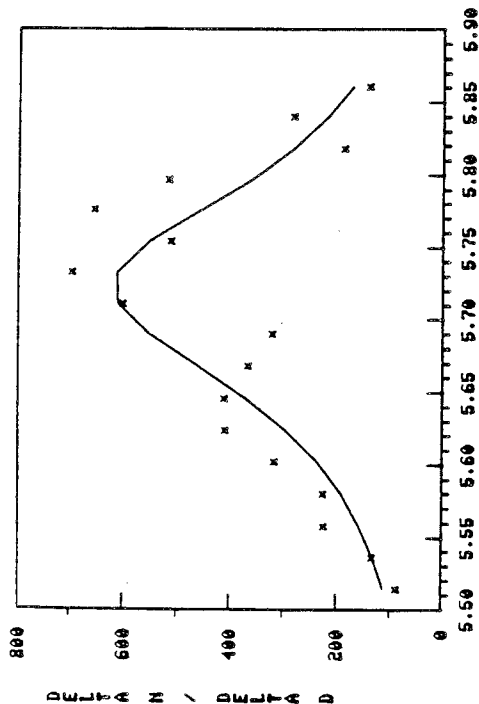
| ITERATION NUMBER | CONVERGENCE MEASURE | RESIDUAL STANDARD DEVIATION | PARAMETER ESTIMATES |
|---------------------|------------------------|-----------------------------------|------------------------|
|---------------------|------------------------|-----------------------------------|------------------------|

| | | | | | | |
|-----|----------|----------|-----------|------------|------------|-----------|
| 1-- | .1000-01 | .1167+03 | .83235+00 | -.11807+00 | -.34921+00 | .30495-01 |
| 2-- | .5765+00 | .1159+03 | .83199+00 | -.11810+00 | -.34923+00 | .30498-01 |
| 3-- | .2919+01 | .1157+03 | .83209+00 | -.11808+00 | -.34923+00 | .30498-01 |
| 4-- | .2499+02 | .1157+03 | .83209+00 | -.11808+00 | -.34923+00 | .30498-01 |

| FINAL PARAMETER ESTIMATES | | | (APPROX. ST. DEV.) |
|---------------------------|------------|--|--------------------|
| 1 A0 | .832090 | | (169.9) |
| 2 A1 | -.118081 | | (24.10) |
| 3 B1 | -.349229 | | (4.68) |
| 4 B2 | .304979-01 | | (.2350-01) |

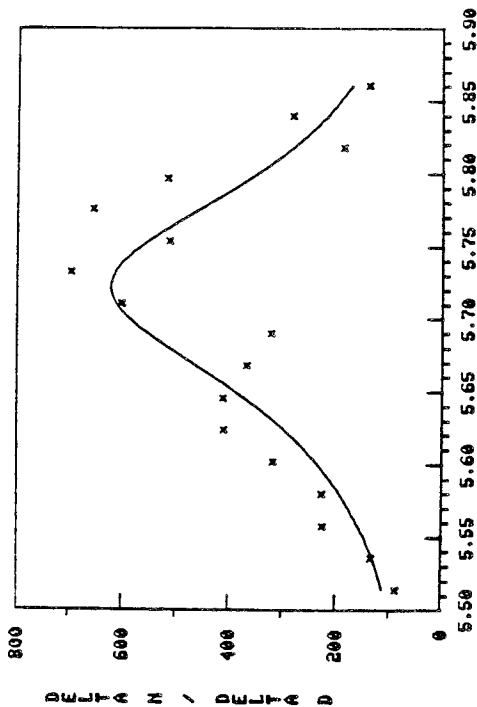
| | | |
|----------|----------------------|----------------|
| RESIDUAL | STANDARD DEVIATION = | 115.7529821396 |
| RESIDUAL | DEGREES OF FREEDOM = | 13 |

PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



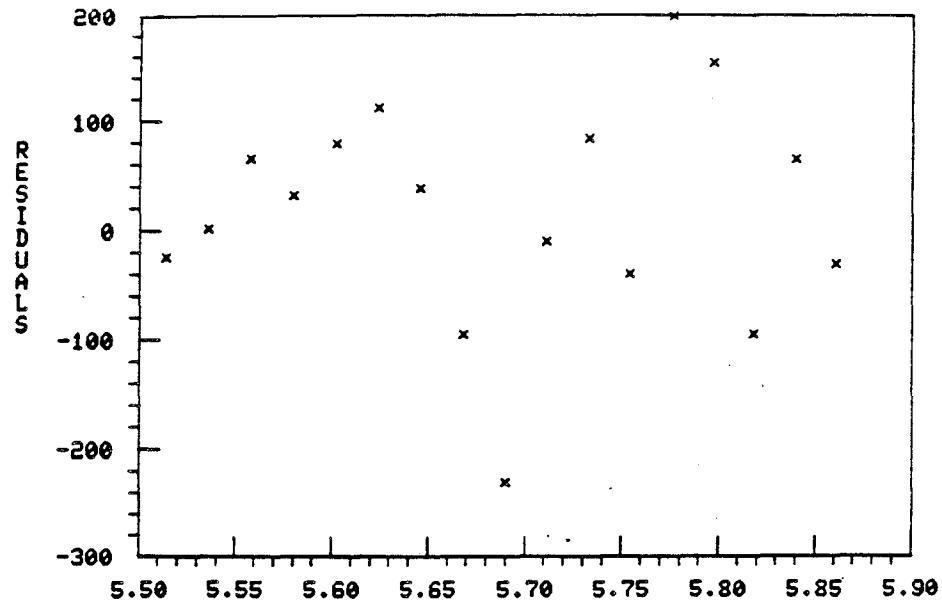
MODEL--Y = (A0 + A1X) / (1 + B1X + B2XX)
JUF68CS9.NONLINEAR30 JUF68DATA.MULHOLLAND2 7/78

PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



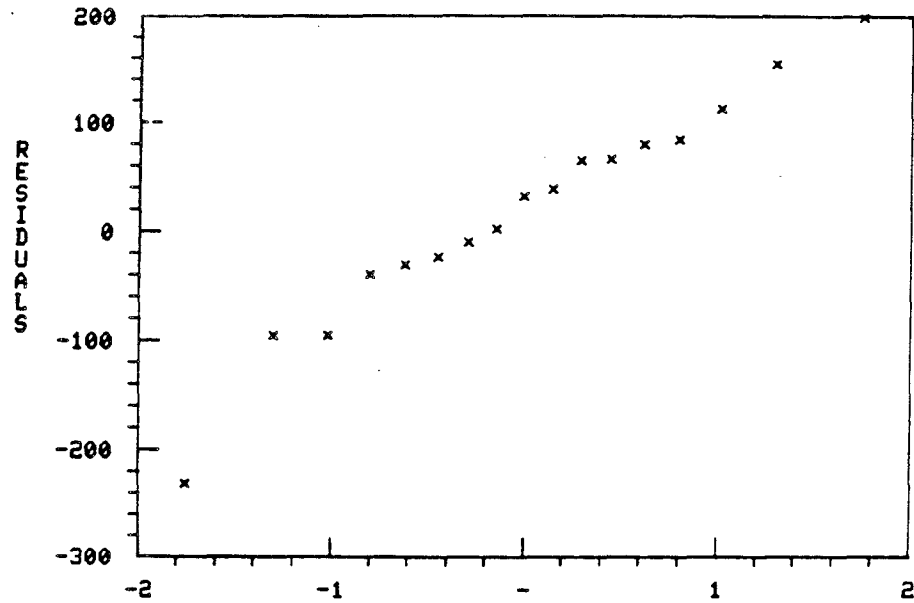
MODEL--Y = (A0 + A1X) / (1 + B1X + B2XX)
JUF68CS9.NONLINEAR30 JUF68DATA.MULHOLLAND2 7/78

PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



DIAMETER
 MODEL--Y = (A0 + A1XX) / (1 + B1XX + B2XXX)
 JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78

PARTICLE SIZE DISTRIBUTION (DOPPLER SPECTROMETER)



DIAMETER
 MODEL--Y = (A0 + A1XX) / (1 + B1XX + B2XXX)
 JJF6*CS9.NONLINEAR30 JJF6*DATA.MULHOLLAND2 7/78

EXAMPLE 24

COMMENT EXAMPLE--NEWT BREESE RESIDENTIAL TIME-TEMPERATURE CURVE
 COMMENT MODEL --CUBIC/CUBIC
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT

ECHO ON

HARDCOPY ON

BELL ON

READ JJF6*DATA.BREESE2

NAME TIME 1

NAME TEMP 2

CHARACTERS X

LINES

PLOT TEMP TIME

LET ID = 0*TEMP

LET ID(1)=1

LET ID(13)=1

LET ID(6)=1

LET ID(11)=1

LET ID(18)=1

LET ID(8)=1

LET ID(21)=1

LET TEMP2 = TEMP SUBSET ID 1

LET TIME2 = TIME SUBSET ID 1

PACK TEMP2 TIME2 SUBSET ID 1

EXACT 3/3 RATIONAL FIT TEMP2 TIME2 TEMP TIME

TITLE DERIVED FIRE EXPOSURE CURVE FIT (T=0,90)

YLABEL TEMPERATURE

X1LABEL TIME (S)

X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT

X3LABEL JJF6*CS9.NONLINEAR21 JJF6*DATA.BREESE2 10/27/78

CHARACTER X BLANK

LINE BLANK SOLID

PLOT TEMP PRED VS TIME

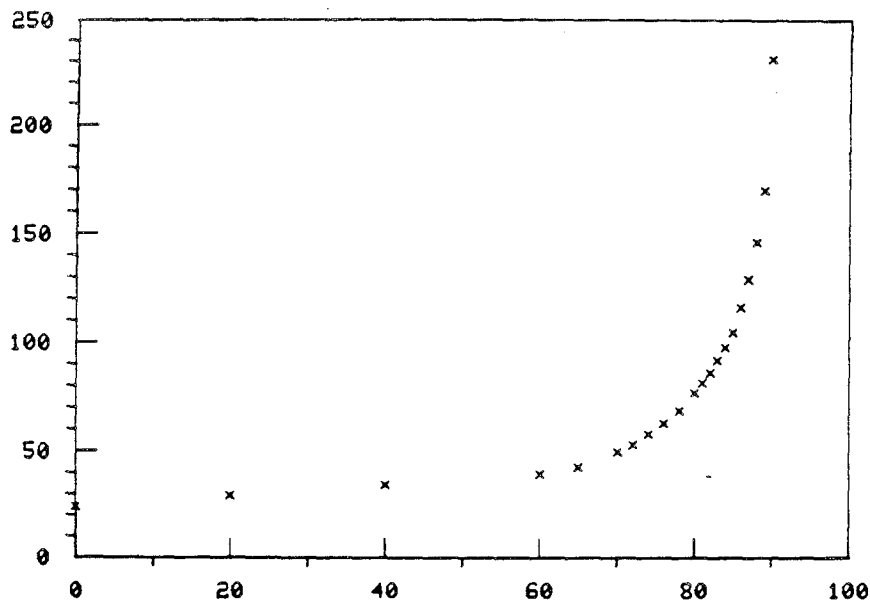
LET A=MIN(TIME)

LET B=MAX(TIME)

LET DEL = (B-A)/100

PLOT TEMP TIME AND

PLOT Y = (A0+A1*X+A2*X*X+A3*X*X*X)/(1+B1*X+B2*X*X+B3*X*X*X) FOR X = A DEL B



```

NUMBER OF POINTS IN FIRST SET      7
DEGREE OF NUMERATOR                 3
DEGREE OF DENOMINATOR               3

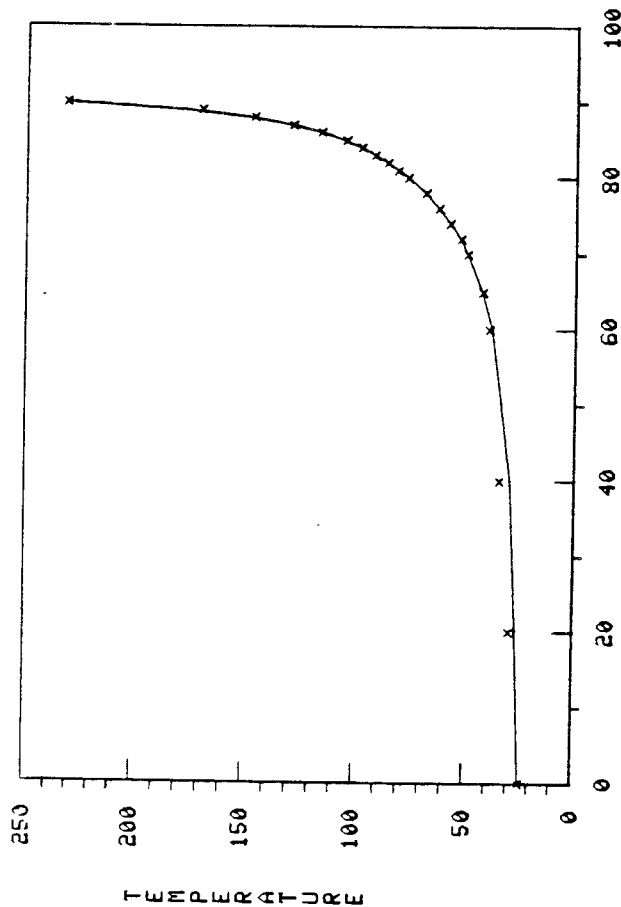
NUMERATOR  --A0 A1 A2 A3      .25000000+02      -.78599218+00      .82258595-02      -.28315299-04
DENOMINATOR --B0 B1 B2 B3      .10000000+01      -.34+18127-01      .39682391-03      -.15308051-05

APPLICATION OF EXACT-FIT COEFFICIENTS
TO SECOND PAIR OF VARIABLES--

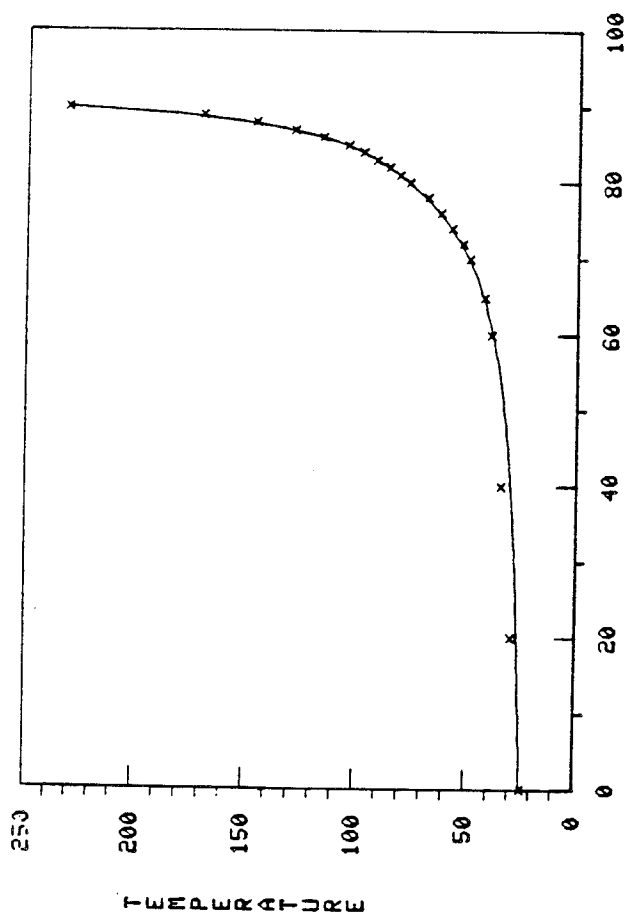
NUMBER OF POINTS IN SECOND SET      21
NUMBER OF ESTIMATED COEFFICIENTS    7
RESIDUAL DEGREES OF FREEDOM         14

RESIDUAL SUM OF SQUARES             .11272736+03
RESIDUAL STANDARD DEVIATION (DENOM=N-P) .28375965+01
AVERAGE ABSOLUTE RESIDUAL (DENOM=N) .112688669+01
LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL .46076736+01
LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL -.85434399+01
LARGEST ABSOLUTE RESIDUAL .85434399+01

```



TIME (S)
EXACT CUBIC/CUBIC RATIONAL FIT
JJF6*CS9.NONLINEAR21 JJF6*DATA.BREESE2 10/27/78



TIME (S)
EXACT CUBIC/CUBIC RATIONAL FIT
JJF6XC9.NONLINEAR21 JJF6XDATA.BREESE2 10/27/78

EXAMPLE 25

```

COMMENT EXAMPLE--PRACTICAL TEMPERATURE SCALE REFERENCE CURVE
COMMENT          FOR LESS THAN 273 DEGREES KELVIN
COMMENT MODEL    --CUBIC/CUBIC
COMMENT NOTE     --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT
COMMENT NOTE     --VALUE OF RESIDUAL PLOT

```

```

ECHO ON
HARDCOPY ON
BELL ON

```

```

READ JJF6*DATA.SCHOOLEY1 T U

```

```

CHARACTERS X
LINES
PLOT T U

```

```

LET ID = 0* $T$ 
LET ID(1)=1
LET ID(5)=1
LET ID(15)=1
LET ID(30)=1
LET ID(70)=1
LET ID(150)=1
LET ID(261)=1
LET T2 = T SUBSET ID 1
LET U2 = U SUBSET ID 1
PACK T2 U2 SUBSET ID 1

```

```

EXACT 3/3 FIT T2 U2 T U

```

```

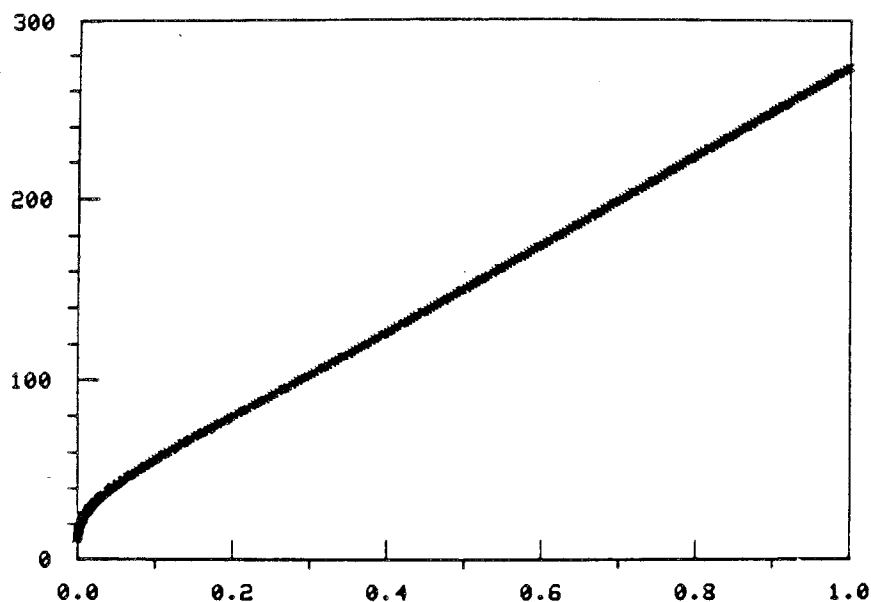
TITLE PRACTICAL TEMPERATURE SCALE REFERENCE CURVE (FOR < 273 K)
YLABEL TEMPERATURE
XLABEL RESISTANCE RATIO U
X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
X3LABEL JJF6*CS9.NONLINEAR22 JJF6*DATA.SCHOOLEY1 10/24/78
CHARACTERS X BLANK
LINES BLANK SOLID
PLOT T PRED US U

```

```

YLABEL RESIDUALS
PLOT RES U

```



EXACT RATIONAL FUNCTION FIT
 NUMBER OF POINTS IN FIRST SET = 7
 DEGREE OF NUMERATOR = 3
 DEGREE OF DENOMINATOR = 3

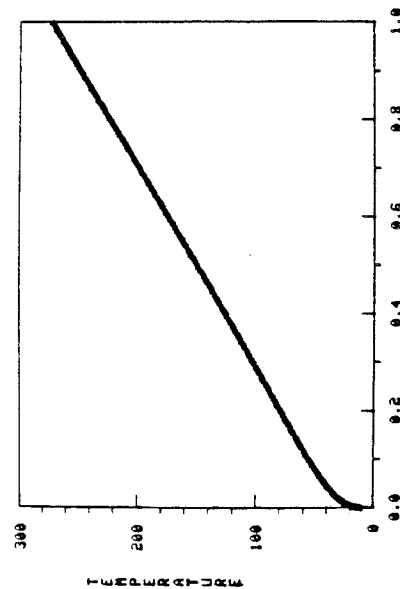
NUMERATOR --A0 A1 A2 A3 .87906122+05 .
 12248801+08 .65114699+08
 DENOMINATOR --B0 B1 B2 B3 .53270117+04 .
 29851157+06 -.20292635+05

APPLICATION OF EXACT-FIT COEFFICIENTS
 TO SECOND PAIR OF VARIABLES--

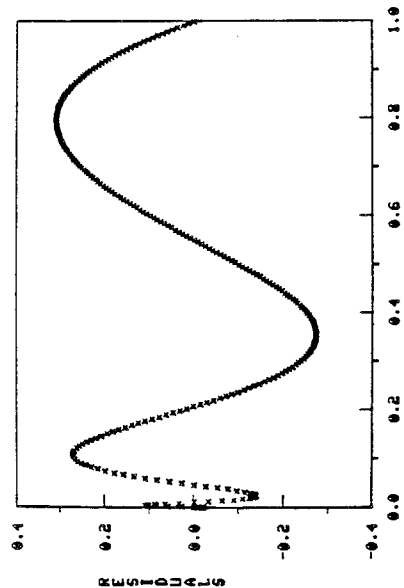
NUMBER OF POINTS IN SECOND SET = 261
 NUMBER OF ESTIMATED COEFFICIENTS = 7
 RESIDUAL DEGREES OF FREEDOM = 254

RESIDUAL SUM OF SQUARES .10105131+02
 RESIDUAL STANDARD DEVIATION (DENOM-N-P) .19945922+00
 AVERAGE ABSOLUTE RESIDUAL (DENOM-N) .17285277+00
 LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL .30939484+00
 LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL -.27073383+00
 LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL .30939484+00

PRACTICAL TEMPERATURE SCALE REFERENCE CURVE (FOR < 273 K)



PRACTICAL TEMPERATURE SCALE REFERENCE CURVE (FOR < 273 K)



EXAMPLE 26

COMMENT EXAMPLE--BOB THURBER SEMICONDUCTOR MOBILITY MODELLING
 COMMENT MODEL --CUBIC/CUBIC
 COMMENT NOTE --EXACT RATIONAL FIT FOLLOWED BY LEAST SQUARES FIT

ECHO ON
 HARDCOPY ON
 BELL ON

READ JJF6*DATA.THURBER22 R N

LET LR=ALOG10(R)
 LET Q = 1.602*10**(-19)
 LET Y = 1/(Q*LR*N)

CHARACTERS X
 LINES
 PLOT Y LR

LET ID=0*N
 LET ID(1) = 1
 LET ID(9) = 1
 LET ID(12) = 1
 LET ID(13) = 1
 LET ID(21) = 1
 LET ID(29) = 1
 LET ID(37) = 1

LET Y2=Y SUBSET ID 1
 LET LR2=LR SUBSET ID 1
 PACK Y2 LR2 SUBSET ID 1

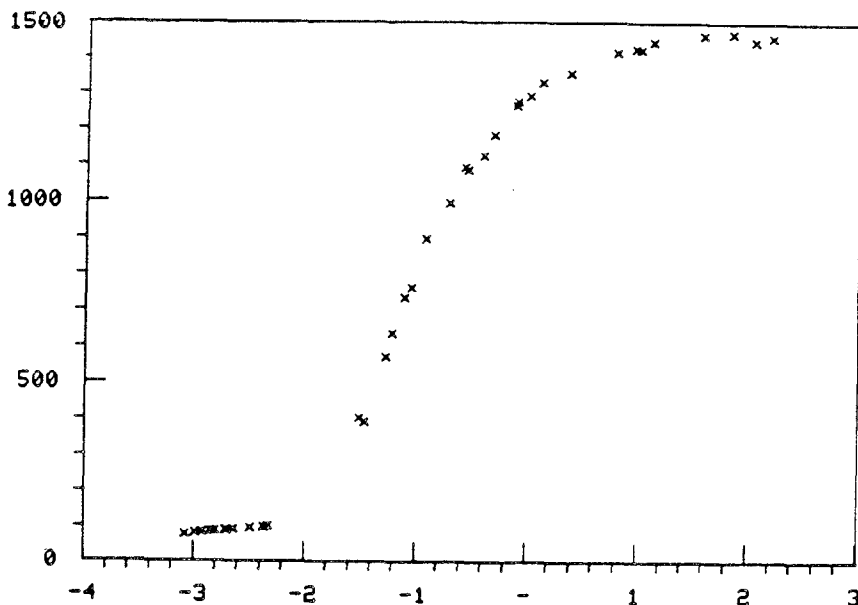
EXACT 3/3 RATIONAL FIT Y2 LR2 Y LR

TITLE PHOSPHORUS-DOPED SILICON 23 C
 YLABEL ELECTRON MOBILITY
 X1LABEL LOG(RHO)
 X2LABEL EXACT CUBIC/CUBIC RATIONAL FIT
 X3LABEL JJF6*CS9.NONLINEAR16 JJF6*DATA.THURBER22 11/7/78
 CHARACTERS X BLANK
 LINES BLANK SOLID
 PLOT Y PRED VS LR

FIT Y = (A0+A1*LR+A2*LR*LR+A3*LR**3)/(1+B1*LR+B2*LR*LR+B3*LR**3)

X2LABEL LEAST SQUARES CUBIC/CUBIC FIT
 PLOT Y PRED VS LR

LET A = MIN(LR)
 LET B = MAX(LR)
 LET INC = (B-A)/100
 PLOT Y LR AND
 PLOT U = (A0+A1*U+A2*U*U+A3*U**3)/(1+B1*U+B2*U*U+B3*U**3) FOR U = A INC B



EXACT RATIONAL FUNCTION FIT

| | | |
|-------------------------------|---|---|
| NUMBER OF POINTS IN FIRST SET | = | 7 |
| DEGREE OF NUMERATOR | = | 3 |
| DEGREE OF DENOMINATOR | = | 3 |

| | | | | | | | | |
|-------------|------|--------------|----|----|---|--------------|--------------|-----|
| NUMERATOR | --A0 | A1 | A2 | A3 | = | .12877952+04 | .14371486+04 | .54 |
| 575087+03 | | .68139794+02 | | | | | | |
| DENOMINATOR | --B0 | B1 | B2 | B3 | = | .10000000+01 | .94009615+00 | .38 |
| 602169+00 | | .40002313-01 | | | | | | |

APPLICATION OF EXACT-FIT COEFFICIENTS TO SECOND PAIR OF VARIABLES--

| | | |
|----------------------------------|---|----|
| NUMBER OF POINTS IN SECOND SET | = | 37 |
| NUMBER OF ESTIMATED COEFFICIENTS | = | 7 |
| RESIDUAL DEGREES OF FREEDOM | = | 30 |

| | | |
|--|---|---------------|
| RESIDUAL SUM OF SQUARES | = | .82915248+04 |
| RESIDUAL STANDARD DEVIATION (DENOM=N-P) | = | .16624805+02 |
| AVERAGE ABSOLUTE RESIDUAL (DENOM=N) | = | .96072464+01 |
| LARGEST (IN MAGNITUDE) POSITIVE RESIDUAL | = | .25774384+02 |
| LARGEST (IN MAGNITUDE) NEGATIVE RESIDUAL | = | -.42453033+02 |
| LARGEST (IN MAGNITUDE) ABSOLUTE RESIDUAL | = | .42453033+02 |

