CHEBU

PURPOSE

Compute the Chebychev polynomial of the second kind and order N.

DESCRIPTION

From Abramowitz and Stegum (see REFERENCE below), a system of nth degree polynomials $f_n(x)$ is called orthognal on the interval a <= x <= b with respect to a weight function w(x) if it satisfies the equation:

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0 \qquad m, n = 0, 1, 2, \ldots, (n \neq m) \qquad \textbf{(EQ Aux-71)}$$

Chebychev polynomials of the second kind use the weight function $(1\text{-}x^2)^{(1/2)}$ and are orthogonal on the interval (-1,1). They are also defined by the following equation:

$$U_n(x) = \frac{\sin((n+1)\arccos(x))}{\sin(\arccos(x))} \qquad -1 \le x \le 1 \qquad \textbf{(EQ Aux-72)}$$

DATAPLOT calculates the Chebychev polynomials using the following recurrence relation:

$$U_n(x) = 2x\, U_{n-1}(x) - U_{n-2}(x) \qquad \textbf{(EQ Aux-73)}$$

where the first few terms for the recuurence were obtained from the Handbook of Mathematical Functions (see the REFERENCE below).

SYNTAX

LET <y> = CHEBU(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable in the range (-1,1);

<n> is a non-negative integer number, parameter, or variable that specifies the order of the Chebychev polynomial;

<y> is a variable or a parameter (depending on what <x> is) where the computed Chebychev polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

EXAMPLES

LET A = CHEBU(-1,4)
LET X2 = CHEBU(X1,10)
LET X2 = CHEBU(X1-0.2,N)

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

CHEBT	=	Compute Chebychev polynomial first kind, order N.
CHEB0	=	Compute Chebychev polynomial first kind, order 0.
CHEB1	=	Compute Chebychev polynomial first kind, order 1.
CHEB2	=	Compute Chebychev polynomial first kind, order 2.
CHEB3	=	Compute Chebychev polynomial first kind, order 3.
CHEB4	=	Compute Chebychev polynomial first kind, order 4.
CHEB5	=	Compute Chebychev polynomial first kind, order 5.
CHEB6	=	Compute Chebychev polynomial first kind, order 6.
CHEB7	=	Compute Chebychev polynomial first kind, order 7.
CHEB8	=	Compute Chebychev polynomial first kind, order 8.
CHEB9	=	Compute Chebychev polynomial first kind, order 9.
CHEB10	=	Compute Chebychev polynomial first kind, order 10.

REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

APPLICATIONS

Function approximation

IMPLEMENTATION DATE

95/7

PROGRAM

```
XLIMITS -1 1; XTIC OFFSET 0.1 0.1
YLIMITS -1 1; YTIC OFFSET 0.1 0.1
LABEL CASE ASIS
TITLE AUTOMATIC
Y1LABEL Tn(X)
X1LABEL X
MULTIPLOT 2 2;;MULTIPLOT CORNER COORDINATES 0 0 100 100
PLOT CHEBU(X,4) FOR X = -1 0.01 1
PLOT CHEBU(X,10) FOR X = -1 0.01 1
PLOT CHEBU(X,20) FOR X = -1 0.01 1
PLOT CHEBU(X,50) FOR X = -1 0.01 1
END OF MULTIPLOT
```