DN

PURPOSE

Compute the Jacobi elliptic function dn.

DESCRIPTION

The Jacobi elliptic function dn is defined as:

$$\mathrm{dn}(u, k) = \sqrt{1 - k\sin(\phi)\sin(\phi)} \qquad \textbf{(EQ Aux-98)}$$

where ϕ is the amplitude and is defined as:

$$u = \int_0^{\phi} \frac{1}{\sqrt{(1 - k\sin(\theta)\sin(\theta))}} d\theta \qquad \textbf{(EQ Aux-99)}$$

The algorithm DATAPLOT uses takes MC=1-k^2 as its second argument rather than k. Be aware that other routines take k^2 as the second argument (e.g., IMSL, NAG, and mathematica). If you want to use k^2, enter something like the following:

```
LET K2 = <value>
LET MC = 1 - K2
```

SYNTAX

LET <a> = DN(<u>,<mc>) <SUBSET/EXCEPT/FOR qualification>

where <u> is a number, parameter, or variable;

<mc> is a number, parameter, or variable;

<a> is a variable or a parameter (depending on what <u> and <mc> are) where the computed values are stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

EXAMPLES

```
LET A = DN(2,1)
LET A = DN(X,0.5)
LET X2 = DN(9,0)
```

NOTE 1

The Jacobi elliptic functions are computed using a Fortran translation of the Algol-60 procedure given by Bulirsch (see the REFERENCE section below).

NOTE 2

DATAPLOT computes the Jacobi elliptic functions sn, cn, and dn. An additional 9 functions can be computed from these:

```
cd(u,k)  =  cn(u,k)/dn(u,k)
sd(u,k)  =  sn(u,k)/dn(u,k)
nd(u,k)  =  1/dn(u,k)
dc(u,k)  =  dn(u,k)/cn(u,k)
nc(u,k)  =  1/cn(u,k)
sc(u,k)  =  sn(u,k)/cn(u,k)
ns(u,k)  =  1/sn(u,k)
ds(u,k)  =  dn(u,k)/sn(u,k)
cs(u,k)  =  cn(u,k)/sn(u,k)
```

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

CN	=	Compute the Jacobi elliptic function cn.
SN	=	Compute the Jacobi elliptic function sn.

RF	=	Compute the Carlson elliptic integral of the first kind.
RD	=	Compute the Carlson elliptic integral of the second kind.
RJ	=	Compute the Carlson elliptic integral of the third kind.
ELLIP1	=	Compute the Legendre elliptic integral of the first kind.
ELLIP2	=	Compute the Legendre elliptic integral of the second kind.
ELLIP3	=	Compute the Legendre's elliptic integral of the third kind.

REFERENCE

"Numerical Calculation of Elliptic Integrals and Elliptic Functions," Bulirsch, Numerische Mathematik, vol. 7, 1965 (pp. 78-90).

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 16).

APPLICATIONS

Special Functions

IMPLEMENTATION DATE

94/11

PROGRAM

```
MULTIPLOT 3 3; MULTIPLOT CORNER COORDINATES 0 0 100 100
LET MC = 0; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = 0.5; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = -0.5; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = 1; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = -1; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = 2; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = -2; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = 5; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
LET MC = -5; TITLE MC = ^MC; PLOT DN(X,MC) FOR X = -10 0.1 10
END OF MULTIPLOT
```