

**ELLIP3****PURPOSE**

Compute Legendre's elliptic integral of the third kind.

**DESCRIPTION**

Legendre's elliptic integral of the third kind is defined as:

$$F(\phi, n, \alpha) = \int_0^{\phi} \frac{1}{(1 + n \sin(\Phi) \sin(\alpha))(\sqrt{1 - \sin(\alpha)^2 \sin(\Phi) \sin(\Phi)})} d\Phi \quad (\text{EQ Aux-114})$$

This function is often expressed as:

$$F(\phi, n, \alpha) = \int_0^{\phi} \frac{1}{(1 + n \sin(\Phi) \sin(\Phi))(\sqrt{1 - k^2 \sin(\Phi) \sin(\Phi)})} d\Phi \quad (\text{EQ Aux-115})$$

That is, k is the sine of  $\alpha$ . In DATAPLOT, the input values are the angles (i.e., the first form) given in radian units. The first angle must be in the interval  $(0, \pi/2)$ .

**SYNTAX**

LET <y> = ELLIP3(<phi>, <n>, <alpha>) <SUBSET/EXCEPT/FOR qualification>  
 where <phi> is a number, variable, or parameter containing values in the interval  $(0, \pi/2)$  exclusive;  
 <n> is a number, variable, or parameter;  
 <alpha> is a number, variable, or parameter;  
 <y> is a variable or a parameter (depending on what <phi> and <alpha> are where the computed values are stored;  
 and where the <SUBSET/EXCEPT/FOR qualification> is optional.

**EXAMPLES**

```
LET A = ELLIP3(0.7,0.5,1.1)
LET A = ELLIP3(0.1,0.2,1.4)
LET X2 = ELLIP3(X,1.0,0.3)
```

**NOTE 1**

If your angles are in degrees, be sure to convert them to radians first. For example, to convert a 45 degree angle to radians, do:

```
LET PHI = 45*(180/PI)
```

Also, if your third parameter is in the form of  $\sin(\alpha)$ , be sure to convert back to the original angle (you can use the ARCSIN function for this).

**NOTE 2**

DATAPLOT computes Legendre elliptic integrals by computing the equivalent Carlson elliptic integrals. The Carlson elliptic integrals are computed using the RC, RD, RF, and RJ routines from the SLATEC Common Mathematical Library to compute this function. SLATEC is a large set of high quality, portable, public domain Fortran routines for various mathematical capabilities maintained by seven federal laboratories.

**DEFAULT**

None

**SYNONYMS**

None

**RELATED COMMANDS**

ELLIPC1	=	Compute the Legendre complete elliptic integral of the first kind.
ELLIPC2	=	Compute the Legendre complete elliptic integral of the second kind.
ELLIP1	=	Compute the Legendre elliptic integral of the first kind.
ELLIP2	=	Compute the Legendre elliptic integral of the second kind.

RC	=	Compute the degenerate Carlson elliptic integral.
RD	=	Compute the Carlson elliptic integral of the second kind.
RF	=	Compute the Carlson elliptic integral of the first kind.
RJ	=	Compute the Carlson elliptic integral of the third kind.

## REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 17).

"Numerical Recipes in FORTRAN," 2nd. Edition, Press, Teukolsky, Vetterling, and Flannery, Cambridge University Press, 1992 (chapter 6).

## APPLICATIONS

Special Functions

## IMPLEMENTATION DATE

94/9

## PROGRAM

```
TITLE AUTOMATIC
LET ALPHA = PI/4
X1LABEL PHI
X2LABEL ALPHA = ^ALPHA
PLOT ELLIP3(X,0.5,ALPHA) FOR X = 0.01 0.01 1.57
```

