HERMITE

PURPOSE

Compute the Hermite polynomial of order N.

DESCRIPTION

From Abramowitz and Stegum (see REFERENCE below), a system of nth degree polynomials $f_n(x)$ is called orthognal on the interval a <= x <= b with respect to a weight function w(x) if it satisfies the equation:

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0 \qquad m, n = 0, 1, 2, \ldots, (n \neq m) \qquad \textbf{(EQ Aux-188)}$$

Hermite polynomials use the weight function EXP(-x^2) and are orthogonal for all real x. They are also defined by the following equation:

$$\mathrm{H}_n(x) = n! \sum_{m=0}^{[n/2]} \frac{(-1)^m (x)^{n-2m}}{m!(n-2m)!} \qquad \textbf{(EQ Aux-189)}$$

where [] represents the integer portion of a number.

DATAPLOT calculates the Hermite polynomials using the following recurrence relation:

$$\mathrm{H}_n(x) = 2x\,\mathrm{H}_{n-1}(x) - 2n\,\mathrm{H}_{n-2}(x) \qquad \textbf{(EQ Aux-190)}$$

where the first few terms for the recuurence were obtained from the Handbook of Mathematical Functions (see the REFERENCE below).

SYNTAX 1:

LET <y> = HERMITE(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable;

<n> is a non-negative integer number, parameter, or variable that specifies the order of the Hermite polynomial;

<y> is a variable or a parameter (depending on what <x> is) where the computed Hermite polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

SYNTAX 2:

LET <y> = LNHERMIT(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable;

<n> is a non-negative integer number, parameter, or variable that specifies the order of the Hermite polynomial;

<y> is a variable or a parameter (depending on what <x> is) where the computed Hermite polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This form of the command computes the logarithm of the absolute value of the Hermite polynomial. It is included primarily for use in intermediate calculations to avoid overflow problems. It is typically used with the HERMSGN function.

SYNTAX 3:

LET <y> = HERMSGN(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable;

<n> is a non-negative integer number, parameter, or variable that specifies the order of the Hermite polynomial;

<y> is a variable or a parameter (depending on what <x> is) where the computed Hermite polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This form of the command computes the sign of the Hermite polynomial. It is included primarily for use in intermediate calculations to avoid overflow problems. It is typically used with the HERMSGN function. It returns a 1 if the Hermite polynomial is positive, a -1 if the Hermite polynomial is negative, and a 0 if the Hermite polynomial is 0.

EXAMPLES

LET A = HERMITE(-1,4)
LET X2 = HERMITE(X1,10)

LET X2 = HERMITE(X1-0.2,N)

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

CHEBT	=	Compute the Chebychev polynomial first kind, order N.
CHEBU	=	Compute the Chebychev polynomial second kind, order N.
JACOBIP	=	Compute the Jacobi polynomial of order N.
LAGUERRE	=	Compute the Laguerre polynomial of order N.
ULTRASPH	=	Compute the ultrasperical polynomial of order N.
LEGENDRE	=	Compute the Legendre polynomial of order N.

REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

APPLICATIONS

Mathematics

IMPLEMENTATION DATE

95/7

PROGRAM

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TITLE SIZE 3; TITLE CASE ASIS; LABEL CASE ASIS
LINE SOLID DASH DOT DASH2
X1LABEL X
MULTIPLOT 2 2; MULTIPLOT CORNER COORDINATES 0 0 100 100
TITLE Hermite Polynomials (order 2 thru 5); Y1LABEL Hn(x)
YMINIMUM 0; YTIC OFFSET 100 0
PLOT HERMITE(X,2) FOR X = 0 .01 3 AND
PLOT HERMITE(X,3) FOR X = 0 .01 3 AND
PLOT HERMITE(X,4) FOR X = 0 .01 3 AND
PLOT HERMITE(X,5) FOR X = 0 .01 3
TITLE Scaled Hermite Polynomials (order 2 thru 5); Y1LABEL Hn(x)/n**3
YLIMITS 0 30; YTIC OFFSET 2 2
PLOT HERMITE(X,2)/(2**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X,3)/(3**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X,4)/(4**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X,5)/(5**3) FOR X = 0 .01 3
YLIMITS; YTIC OFFSET 0 0
TITLE Scaled Hermite Polynomials (order 2 thru 5); Y1LABEL Hn(x/SQRT(2))
PLOT HERMITE(X/SQRT(2),2)/(2**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X/SQRT(2),3)/(3**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X/SQRT(2),4)/(4**3) FOR X = 0 .01 3 AND
PLOT HERMITE(X/SQRT(2),5)/(5**3) FOR X = 0 .01 3
TITLE Weber functions (order 2 thru 5); Y1LABEL EXP(-x**2/4)*Hn(x/SQRT(2))
LET FUNCTION F = EXP(-X**2)*HERMITE(X/SQRT(2),N)
LET N = 2; PLOT F FOR X = -5 .01 5 AND
LET N = 3; PLOT F FOR X = -5 .01 5 AND
LET N = 4; PLOT F FOR X = -5 .01 5 AND
LET N = 5; PLOT F FOR X = -5 .01 5
END OF MULTIPLOT
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