JACOBIP

PURPOSE

Compute the Jacobi polynomial of order N.

DESCRIPTION

From Abramowitz and Stegum (see REFERENCE below), a system of nth degree polynomials $f_n(x)$ is called orthognal on the interval a <= x <= b with respect to a weight function w(x) if it satisfies the equation:

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0 \qquad m, n = 0, 1, 2, \ldots, (n \neq m) \tag{EQ Aux-208}$$

Jacobi polynomials use the weight function $(1-x)^{\alpha}(1+x)^{\beta}$, where α and β are shape parameters both > -1, and are orthogonal for -1 <= x <= 1. Jacobi polynomials can also be defined by the following equation:

$$P_n^{\alpha, \beta}(x) = \frac{1}{2^n} \sum_{m=0}^{n} (-1)^m \binom{n+\alpha}{m} \binom{n+\beta}{n-m} (x-1)^{n-m} (x+1)^m \tag{EQ Aux-209}$$

DATAPLOT uses ACM algorithm 332 with suggestions given in the Remark on Algorithm 332 (see Reference section below) to calculate the Jacobi polynomials. This algorithm computes Jacobi polynomials for orders 0 to 25. An error message is printed if the requested degree exceeds 25.

SYNTAX:

LET <y> = JACOBIP(<x>,<n>,<a>,<b>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable in the range (-1,1);

<n> is a non-negative integer number, parameter, or variable that specifies the order of the JACOBIP polynomial;

<a> is a number, parameter, or variable that specifies the first shape parameter;

<b> is a number, parameter, or variable that specifies the second shape parameter;

<y> is a variable or a parameter (depending on what <x> is) where the computed Jacobi polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

EXAMPLES

LET A = JACOBIP(-0.5,4,2.5,3)
LET X2 = JACOBIP(X1,N,A,B)

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

CHEBT	=	Compute the Chebychev polynomial first kind, order N.
HERMITE	=	Compute the Hermite polynomial of order N.
LAGUERRE	=	Compute the Laguerre polynomial of order N.
ULTRASPH	=	Compute the ultrasperical polynomial of order N.
LEGENDRE	=	Compute the Legendre polynomial of order N.

REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

"Algorithm 332: Jacobi Polynomials," Witte, Communication of the ACM, Vol. 11, June, 1968 (page 436).

"Remark on Algorithm 332," Skivgaard, Communication of the ACM, Vol. 18, February, 1975 (pp. 116-117).

APPLICATIONS

Mathematics

IMPLEMENTATION DATE

95/7

PROGRAM

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TITLE CASE ASIS; LABEL CASE ASIS; LINE SOLID DASH DOT DASH2
TITLE Jacobi polynomials (order 1 thru 5); Y1LABEL Jn(X,a,b); X1LABEL X
MULTIPLOT 2 2; MULTIPLOT CORNER COORDINATES 0 0 100 100
LET ALPHA = 1.5; LET BETA = -0.5; X2LABEL ALPHA = ^ALPHA, BETA = ^BETA
PLOT JACOBIP(X,1,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,2,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,3,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,4,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,5,ALPHA,BETA) FOR X = -0.9 .01 0.9
LET ALPHA = 0.5; LET BETA = 0.5; X2LABEL ALPHA = ^ALPHA, BETA = ^BETA
PLOT JACOBIP(X,1,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,2,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,3,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,4,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,5,ALPHA,BETA) FOR X = -0.9 .01 0.9
LET ALPHA = 2; LET BETA = 3; X2LABEL ALPHA = ^ALPHA, BETA = ^BETA
PLOT JACOBIP(X,1,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,2,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,3,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,4,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,5,ALPHA,BETA) FOR X = -0.9 .01 0.9
LET ALPHA = 10; LET BETA = 0.5; X2LABEL ALPHA = ^ALPHA, BETA = ^BETA
PLOT JACOBIP(X,1,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,2,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,3,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,4,ALPHA,BETA) FOR X = -0.9 .01 0.9 AND
PLOT JACOBIP(X,5,ALPHA,BETA) FOR X = -0.9 .01 0.9; END OF MULTIPLOT
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