LAGUERRE

PURPOSE

Compute the Laguerre, normalized Laguerre, or the generalized Laguerre polynomial of order N.

DESCRIPTION

From Abramowitz and Stegum (see REFERENCE below), a system of nth degree polynomials $f_n(x)$ is called orthognal on the interval a <= x <= b with respect to a weight function w(x) if it satisfies the equation:

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0 \qquad m, n = 0, 1, 2, \ldots, (n \neq m) \qquad \textbf{(EQ Aux-216)}$$

Laguerre polynomials use the weight function EXP(-x) and are orthogonal for non-negative x. Laguerre polynomials can also be defined by the following equation:

$$L_n(x) = \sum_{m=0}^{n} \frac{(-1)^m \binom{n}{n-m} x^m}{m!} \qquad \textbf{(EQ Aux-217)}$$

Normalized Laguerre polynomials scale the Laguerre polynomials as follows:

$$NL_n(x) = n!\, L_n(x) \qquad \textbf{(EQ Aux-218)}$$

Generalized Laguerre polynomials use the weight function $x^{\alpha}e^{(-x)}$ and are orthogonal for non-negative x. The value of $\alpha > -1$. The generalized Laguerre polynomial can also be defined by the following equation:

$$L_n^{\alpha}(x) = \sum_{m=0}^{n} \frac{(-1)^m \binom{n+\alpha}{n-m} x^m}{m!} \qquad \textbf{(EQ Aux-219)}$$

DATAPLOT calculates the Laguerre polynomials using the following recurrence relation:

$$L_n(x) = \frac{(((2n+1)-x)\, L_{n-1}(x) - n\, L_{n-2}(x))}{n+1} \qquad \textbf{(EQ Aux-220)}$$

and the normalized Laguerre polynomials with the following recurrence relation:

$$NL_n(x) = (1+2n-x)\, NL_{n-1}(x) - n^2\, NL_{n-2}(x) \qquad \textbf{(EQ Aux-221)}$$

and the generalized Laguerre polynomials with the following recurrence relation:

$$L_n(x,a) = \frac{(((2n+a+1)-x)\, L_{n-1}(x,a) - (n+a)\, L_{n-2}(x,a))}{n+1} \qquad \textbf{(EQ Aux-222)}$$

where the first few terms for the recuurence were obtained from the Handbook of Mathematical Functions (see the REFERENCE below).

SYNTAX 1:

LET <y> = LAGUERRE(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a non-negative number, parameter, or variable;

<n> is a non-negative integer number, parameter, or variable that specifies the order of the Laguerre polynomial;

<y> is a variable or a parameter (depending on what <x> is) where the computed Laguerre polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes Laguerre polynomials.

SYNTAX 2:

LET <y> = NRMLAG(<x>,<n>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a non-negative number, parameter, or variable;
<n> is a non-negative integer number, parameter, or variable that specifies the order of the Laguerre polynomial;
<y> is a variable or a parameter (depending on what <x> is) where the computed Laguerre polynomial value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes normalized Laguerre polynomials.

SYNTAX 3:

LET <y> = LAGUERRL(<x>,<n>,<a>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a non-negative number, parameter, or variable;
<n> is a non-negative integer number, parameter, or variable that specifies the order of the Laguerre polynomial;
<a> is a number, parameter, or variable that specifies the shape parameter;
<y> is a variable or a parameter (depending on what <x> is) where the computed Laguerre polynomial value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes generalized Laguerre polynomials.

EXAMPLES

LET A = LAGUERRE(0.5,4,2.5)
LET X2 = LAGUERRE(X1,10,0.5)
LET X2 = LAGUERRE(X1,N,A)

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

CHEBT	=	Compute the Chebychev polynomial first kind, order N.
CHEBU	=	Compute the Chebychev polynomial second kind, order N.
HERMITE	=	Compute the Hermite polynomial of order N.
JACOBIPE	=	Compute the Jacobi polynomial of order N.
ULTRASPH	=	Compute the ultrasperical polynomial of order N.
LEGENDRE	=	Compute the Legendre polynomial of order N.

REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

APPLICATIONS

Mathematics

IMPLEMENTATION DATE

95/7

PROGRAM

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TITLE CASE ASIS; LABEL CASE ASIS
LINE SOLID DASH DOT DASH2; X1LABEL X
.
MULTIPLOT 2 2; MULTIPLOT CORNER COORDINATES 0 0 100 100
TITLE Laguerre Polynomials (order 2 thru 5); Y1LABEL Ln(X)
PLOT LAGUERRE(X,2) FOR X = 0 .01 5 AND
PLOT LAGUERRE(X,3) FOR X = 0 .01 5 AND
PLOT LAGUERRE(X,4) FOR X = 0 .01 5 AND
PLOT LAGUERRE(X,5) FOR X = 0 .01 5
.
TITLE Normalized Laguerre Polynomials (order 2 thru 5); Y1LABEL NLn(X)
PLOT NRMLAG(X,2) FOR X = 0 .01 5 AND
PLOT NRMLAG(X,3) FOR X = 0 .01 5 AND
PLOT NRMLAG(X,4) FOR X = 0 .01 5 AND
PLOT NRMLAG(X,5) FOR X = 0 .01 5
.
TITLE Generalized Laguerre Polynomials (order 2 thru 5); Y1LABEL Ln(X,^a)
LET A = 3; X2LABEL A = ^A
PLOT LAGUERRL(X,2,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,3,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,4,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,5,A) FOR X = 0 .01 5
.
LET A = 0.5; X2LABEL A = ^A; Y1LABEL Ln(X,^a)
PLOT LAGUERRL(X,2,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,3,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,4,A) FOR X = 0 .01 5 AND
PLOT LAGUERRL(X,5,A) FOR X = 0 .01 5
END OF MULTIPLOT
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