SPHRHRMR

PURPOSE

Compute the spherical harmonic function of order N and degree M.

DESCRIPTION

The spherical harmonic function is related to the associated Legendre function as follows:

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}}\, P_n^m(\cos(\theta))e^{im\phi} \qquad \textbf{(EQ Aux-308)}$$

where $P_{n,m}$ is the associated Legendre polynomial. See the documentation for the LEGENDRE command for a description of Legendre polynomials and associated Legendre polynomials. DATAPLOT use the NORMP set of routines from the Slatec library to compute the normalized associated Legendre polynomials. These are then transformed in an appropriate manner to obtain the spherical harmonic value. These routines use a technique called extended range arithmetic to avoid underflow and overflow problems. However, DATAPLOT stores the result as a single precision real number. If it is unable to do so, it prints an error message.

SYNTAX 1

LET <y> = SPHRHRMR(<theta>,<phi>,<n>,<m>) <SUBSET/EXCEPT/FOR qualification>
where <theta> is a number, parameter, or variable in the range (-PI,PI);
<phi> is a number, parameter, or variable in the range (-PI,PI);
<n> is a non-negative integer number, parameter, or variable that specifies the order of the Legendre polynomial;
<m> is a non-negative integer number, parameter, or variable that specifies the degree of the Legendre polynomial;
<y> is a variable or a parameter (depending on what <x> is) where the computed spherical harmonic value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the real component of the spherical harmonic function.

SYNTAX 2

LET <y> = SPHRHRMC(<theta>,<phi>,<n>,<m>) <SUBSET/EXCEPT/FOR qualification>
where <theta> is a number, parameter, or variable in the range (-PI,PI);
<phi> is a number, parameter, or variable in the range (-PI,PI);
<n> is a non-negative integer number, parameter, or variable that specifies the order of the Legendre polynomial;
<m> is a non-negative integer number, parameter, or variable that specifies the degree of the Legendre polynomial;
<y> is a variable or a parameter (depending on what <x> is) where the computed spherical harmonic value is stored;
and where the <SUBSET/EXCEPT/FOR qualification> is optional.

This syntax computes the complex component of the spherical harmonic function.

EXAMPLES

DEGREES; LET AR = SPHRHRMR(45,60,5,2); LET AC = SPHRMRC(45,60,5,2)
DEGREES; LET AR = SPHRHRMR(T,P,N,M); LET AC = SPHRMRC(T,P,N,M)

NOTE

Spherical harmonics are often specified with an angular input value. If your input value is in terms of the (-1,1) interval, use the command

LET TNEW = ARCCOS(T); LET PNEW = ARCCOS(P)

and then use TNEW and PNEW as the input arguments to the spherical harmonic functions.; By default, the angle is specified in radians. Enter the command DEGREES to specify degree units.

DEFAULT

None

SYNONYMS

None

RELATED COMMANDS

LEGENDRE = Compute the Legendre polynomial of order N.

REFERENCE

"Extended-Range Arithmetic and Normalized Legendre Polynomials," Smith, Olver, and Lozier, ACM Transactions On Mathematical Software, Vol. 7, No. 1, March, 1981 (pp. 93-105).

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

APPLICATIONS

Mathematics

IMPLEMENTATION DATE

95/7

PROGRAM

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TITLE CASE ASIS; LABEL CASE ASIS; LINE SOLID DASH; DEGREES
Y1LABEL Y(theta,phi,n,m); X1LABEL THETA   (phi = 60, n = 5)
X2LABEL SOLID = Real Component; X3LABEL DASH = Complex Component
TITLE Spherical Harmonics (m = 2)
MULTIPLOT 2 2; MULTIPLOT CORNER COORDINATES 0 0 100 100
PLOT SPHRHRMR(X,60,5,2) FOR X = 1 1 89 AND
PLOT SPHRHRMC(X,60,5,2) FOR X = 1 1 89
TITLE Spherical Harmonics (m = 3)
PLOT SPHRHRMR(X,60,5,3) FOR X = 1 1 89 AND
PLOT SPHRHRMC(X,60,5,3) FOR X = 1 1 89
TITLE Spherical Harmonics (m = 4)
PLOT SPHRHRMR(X,60,5,4) FOR X = 1 1 89 AND
PLOT SPHRHRMC(X,60,5,4) FOR X = 1 1 89
TITLE Spherical Harmonics (m = 5)
PLOT SPHRHRMR(X,60,5,5) FOR X = 1 1 89 AND
PLOT SPHRHRMC(X,60,5,5) FOR X = 1 1 89
END OF MULTIPLOT
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