# ULTRASPH

## PURPOSE

Compute the ultraspherical (or Gegenbauer) polynomial of order N.

## DESCRIPTION

From Abramowitz and Stegum (see REFERENCE below), a system of nth degree polynomials $f_n(x)$ is called orthognal on the interval a <= x <= b with respect to a weight function w(x) if it satisfies the equation:

$$\int_a^b w(x) f_n(x) f_m(x) dx = 0 \qquad m, n = 0, 1, 2, \ldots, (n \neq m) \qquad \textbf{(EQ Aux-317)}$$

Ultraspherical polynomials use the weight function $(1-x^2)^{(\alpha-1/2)}$, where $\alpha$ is a shape parameter, and are orthogonal for -1<=x<=1. Ultraspherical polynomials can also be defined by the following equation:

$$C_n^{\alpha}(x) = \frac{1}{\Gamma(\alpha)} \sum_{m=0}^{\left[\frac{n}{2}\right]} \frac{(-1)^m \Gamma(\alpha + n - m)(2x)^{n-2m}}{m!(n-2m)!} \qquad \textbf{(EQ Aux-318)}$$

where $\Gamma$ is the gamma function (see the documentation for the GAMMA command for details), and [] signifies the integer portion, and $\alpha$ is a shape parameter. DATAPLOT uses the following recurrence relation to compute the ultraspherical polynomial:

$$C_n^{\alpha} x = \frac{\Gamma\left(\alpha + \frac{1}{2}\right)\Gamma(2\alpha + n)}{\Gamma\left(\alpha + n + \frac{1}{2}\right)\Gamma(2\alpha)} P_n^{\alpha - \frac{1}{2}, \alpha - \frac{1}{2}}(x) \qquad \textbf{(EQ Aux-319)}$$

where $P_n$ is the Jacobi polynomial (see the documentation for the JACOBI command for details) and $\Gamma$ is the gamma function. DATAPLOT uses ACM algorithm 332 with suggestions given in Remark on Algorithm 332 (see Reference section below) to calculate the Jacobi polynomials. This algorithm calculates Jacobi polynomials for orders 0 to 25. Therefore, the ultraspherical polynomials are also restricted to orders 0 to 25.

## SYNTAX

LET <y> = ULTRASPH(<x>,<n>,<a>) <SUBSET/EXCEPT/FOR qualification>

where <x> is a number, parameter, or variable in the range (-1,1);

<n> is a non-negative integer number, parameter, or variable that specifies the order of the ultraspherical polynomial;

<a> is a number, parameter, or variable that specifies the shape parameter;

<y> is a variable or a parameter (depending on what <x> is) where the computed Laguerre polynomial value is stored;

and where the <SUBSET/EXCEPT/FOR qualification> is optional.

## EXAMPLES

```
LET A = ULTRASPH(0.5,4,2.5)
LET X2 = ULTRASPH(X1,10,0.5)
LET X2 = ULTRASPH(X1,N,A)
```

## DEFAULT

None

## SYNONYMS

None

## RELATED COMMANDS

| | | |
|---|---|---|
| CHEBT | = | Compute the Chebychev polynomial first kind, order N. |
| CHEBU | = | Compute the Chebychev polynomial second kind, order N. |
| HERMITE | = | Compute the Hermite polynomial of order N. |
| JACOBIP | = | Compute the Jacobi polynomial of order N. |
| LAGUERRE | = | Compute the Laguerre polynomial of order N. |

LEGENDRE = Compute the Legendre polynomial of order N.

## REFERENCE

"Handbook of Mathematical Functions, Applied Mathematics Series, Vol. 55," Abramowitz and Stegun, National Bureau of Standards, 1964 (chapter 22).

"Algorithm 332: Jacobi Polynomials," Witte, Communication of the ACM, Vol. 11. June, 1968 (page 436).

"Remark On Algorithm 332," Skivgaard, Communication of the ACM, Vol. 18, February, 1975 (pp. 116-117).

## APPLICATIONS

Mathematics

## IMPLEMENTATION DATE

95/7

PROGRAM

```
TITLE SIZE 3; TITLE CASE ASIS
LABEL CASE ASIS; LINE SOLID DASH DOT DASH2
TITLE Ultraspherical (or Gegenbauer) polynomials (order 1 thru 5); Y1LABEL Cn(X,alpha); X1LABEL X
MULTIPLOT 2 2; MULTIPLOT CORNER COORDINATES 0 0 100 100
LET ALPHA = 0.5; X2LABEL Alpha = ^ALPHA
PLOT ULTRASPH(X,1,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,2,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,3,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,4,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,5,^ALPHA) FOR X = -0.9 .01 0.9
LET ALPHA = 1; X2LABEL Alpha = ^ALPHA
PLOT ULTRASPH(X,1,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,2,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,3,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,4,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,5,^ALPHA) FOR X = -0.9 .01 0.9
LET ALPHA = 2; X2LABEL Alpha = ^ALPHA
PLOT ULTRASPH(X,1,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,2,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,3,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,4,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,5,^ALPHA) FOR X = -0.9 .01 0.9
LET ALPHA = 5; X2LABEL Alpha = ^ALPHA
PLOT ULTRASPH(X,1,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,2,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,3,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,4,^ALPHA) FOR X = -0.9 .01 0.9 AND
PLOT ULTRASPH(X,5,^ALPHA) FOR X = -0.9 .01 0.9
END OF MULTIPLOT
```