

# Mathematica software for numerical simulation of hurricane-borne missiles

## Software manual

In this manual we present the *Mathematica* subroutine used for producing the numerical results from the report "Technical Basis for Regulatory Guidance on Design-Basis Hurricane-Borne Missile Speeds for Nuclear Power Plants" by Emil Simiu and Florian Potra, as well as a high accuracy variant of that subroutine which has enhanced functionality. In order to be able to use the subroutines you must have the software package *Mathematica* 6 installed on your computer. The subroutines are contained in the *Mathematica* notebook **mv.nb**. Examples for using the subroutines are given in the *Mathematica* notebook **examples.nb**. The examples are also discussed in the present manual.

### 1. Installing the subroutines.

- 1.1. Save a copy of the file **mv.nb** in your working directory.
- 1.2. Click on the icon **mv.nb** in order to open the subroutine.
- 1.3. Press **Ctrl-a** in order to select all the cells from **mv.nb**.
- 1.4. Press SHIFT-ENTER in order to initialize the notebook.
- 1.5. At the prompt press **Yes**.

Note: step 1.1 has to be performed only once. Steps 1.1-1.5 have to be performed for each new session.

### 2. The subroutine missileVelocities

The subroutine **missileVelocities** was used to produce the numerical results in the report. The numerical values computed by this subroutine are accurate within the corresponding units of measure (m, m/s, degree). The subroutine can be used for the 6 types of missiles described in the report and is relatively fast. The subroutine is called as

```
missileVelocities[missileType, terrain, V10, h]
```

where the inputs **missileType**, **terrain**, **V10**, **h** are described as follows:

#### ■ 2.1 INPUT

**missileType** defines one of the type of missile we want to simulate.

The following types of missiles are supported :

```
missileType = 1, a = 0.0021 m-1 (solid steel sphere)
missileType = 2, a = 0.0026 m-1 (sch .40 pipe)
missileType = 3, a = 0.0042 m-1 (5 m x 2 m x 1.3 m automobile)
missileType = 4, a = 0.0057 m-1 (4.5 m x 1.7 m x 1.5 m automobile)
missileType = 5, a = 0.0885 m-1 (slab)
missileType = 6, a = 0.1760 m-1 (plank)
missileType = 7, a = 0.079 m-1 (plank)
```

Here **a** is the missile characteristic given by equation (1c) from the report

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**terrain** describes the type of terrain considered. It is related to the parameter  $\alpha$  defined following equation (3c) from the report. Two types of terrain are considered:

```
terrain = 1 corresponds to open terrain ( $\alpha = 2/19$ )  
terrain = 2 corresponds to suburban terrain ( $\alpha = 1/7$ )
```

**v10** is the value of  $v_h^{\text{open}}(10 \text{ m})$ , the 3-s peak gust wind speed in open terrain at 10 m above the ground. (in m/s as defined in the report, can be any positive real number)

**h** is the starting height above the ground of missile (in m, as defined in the report, can be any positive real number)

### ■ 2.2 OUTPUT

The printed output of the subroutine contains:

```
terminal horizontal velocity (in m/s)  
terminal total velocity (in m/s)  
angle of implact with ground (in degrees)  
maximum horizontal velocity (in m/s) and height at which it is attained (in m )  
maximum total velocity (in m/s) and height at which it is attained (in m )
```

### ■ 2.3. Examples of using the subroutine `missileVelocities`

In order to use the subroutine, open a new notebook in *Mathematica* by selecting File>New>Notebook from the Mathematica toolbar. Type into an input cell

```
missileVelocities[1, 2, 40, 40]
```

and press SHIFT-ENTER. The printed output of the subroutine is:

```
missileType = 1, terrain = 2, V10 = 40 m/s, h = 40 m  
terminal horizontal velocity = 7.9 m/s  
terminal total velocity = 27.2 m/s  
angle of implact with ground = 73 degrees  
maximum horizontal velocity = 7.9 m/s attained at a height of 0.02 m  
maximum total velocity = 27.2 m/s attained at a height of 0.02 m
```

Similarly, by typing

```
missileVelocities[5, 2, 150, 30]
```

and pressing SHIFT-ENTER, one gets the output

```

missileType = 5, terrain = 2, V10 = 150 m/s, h = 30 m
terminal horizontal velocity = 101.4 m/s
terminal total velocity = 101.6 m/s
angle of impact with ground = 3 degrees
maximum horizontal velocity = 141.1 m/s attained at a height of 20.46 m
maximum total velocity = 141.4 m/s attained at a height of 20.25 m

```

### 3. The subroutine missileVelocitiesA

The subroutine **missileVelocitiesA** is a more accurate version of the subroutine presented in Section 2, and has extended functionality. It can accept any type of missile, as defined by the parameter  $a$ , the missile characteristic given by equation (1c) from the report. Also, it can be used to get information at any point on the trajectory of the missile, and to plot the quantities of interest along the trajectory..

#### ■ 3.1 Using the subroutine missileVelocitiesA with printed output only

In this mode **missileVelocitiesA** behaves just like **missileVelocities** with the exception that the numerical results are more accurate. In this case the subroutine is called as

```
missileVelocitiesA[a, terrain, V10, h]
```

where  $a$  is the missile characteristic (cf. Eqn (1c) from the report) and  $terrain$ ,  $V10$ ,  $h$  are as defined in subsection 2.1.

The printed output of the subroutine contains:

```

terminal horizontal velocity (in m/s)
terminal total velocity (in m/s)
angle of impact with ground (in degrees)
maximum horizontal velocity (in m/s) and height at which it is attained (in m)
maximum total velocity (in m/s) and height at which it is attained (in m)

```

By typing into an input cell

```
missileVelocitiesA[0.0021, 2, 40, 40]
```

and pressing SHIFT-ENTER one obtains the following printed output.

```

a = 0.0021, terrain = 2, V10 = 40 m/s, h = 40 m
terminal horizontal velocity = 7.92893 m/s
terminal total velocity = 27.2482 m/s
angle of impact with ground = 73.0828 degrees
maximum horizontal velocity = 7.92893 m/s attained at a height of 0.000439053 m
maximum total velocity = 27.2482 m/s attained at a height of  $2.27111 \times 10^{-11}$  m

```

This is to be compared with the results given by the lower accuracy subroutine presented in subsection 2.3.

Similarly, by typing

```
missileVelocitiesA[0.0885, 2, 150, 30]
```

and pressing SHIFT-ENTER one obtains,

```
a = 0.0885, terrain = 2, V10 = 150 m/s, h = 30 m
terminal horizontal velocity = 101.415 m/s
terminal total velocity = 101.584 m/s
angle of implact with ground = 3.30295 degrees
maximum horizontal velocity = 141.069 m/s attained at a height of 20.4574 m
maximum total velocity = 141.359 m/s attained at a height of 20.252 m
```

### ■ 3.2 Using the subroutine `missileVelocitiesA` with extended functionality

When `missileVelocitiesA` is called as

```
{ {vfhA, tifA}, {vftA, afa}, {vhmA, hmA, thmA}, {vtmA, htA,ttmA}, {xhA, xzA, vhA, vzA, at} }
= missileVelocitiesA[a, terrain, V10, h]
```

then the following quantities will be defined in the current *Mathematica* notebook:

```
vfhA = terminal horizontal velocity (in m)
tifA = time of flight, until impact with ground (in s)
vftA = terminal total velocity (in m/s)
afa = angle of implact with ground (in degrees)
vhmA = maximum horizontal velocity (in m/s)
hmA = height at which maximum horizontal velocity is attained (in m)
thmA = height at which maximum horizontal velocity is attained (in s)
vtmA = maximum total velocity (in m/s)
htA = height at which maximum total velocity velocity is attained (in m)
ttmA = height at which maximum total velocity velocity is attained (in s)
xhA = horizontal coordinate of the missile as a function of time (in m)
xvA = vertical coordinate of the missile as a function of time (in m)
vhA = horizontal velocity of the missile as a function of time (in m/s)
vzA = vertical velocity of the missile as a function of time (in m/s)
vtA = tottal velocity of the missile as a function of time (in m/s)
atA = angle between the the total velocity vector and the horinzontal axis as a fuction of time ( in degrees).
```

The latter 5 quantities are functions that can be evaluated at any time  $t$  for any  $t$  between 0 and  $tifA$  and plotted as shown below.

#### ■ 3.2.1 By typing the following instruction in an input cell and intializing it with SHIFT - ENTER we obtain

```
{ {vfhA, tifA}, {vftA, afa}, {vhmA, hmA, thmA}, {vtmA, htA, ttmA},
  {xhA, xvA, vhA, vzA, vtA, atA} } = missileVelocitiesA[0.0021, 2, 40, 40];
```

```

a = 0.0021, terrain = 2, V10 = 40 m/s, h = 40 m
terminal horizontal velocity = 7.92893 m/s
terminal total velocity = 27.2482 m/s
angle of impact with ground = 73.0828 degrees
maximum horizontal velocity = 7.92893 m/s attained at a height of 0.000439053 m
maximum total velocity = 27.2482 m/s attained at a height of  $2.27111 \times 10^{-11}$  m

```

In what follows it is assumed that 3.2.1 has already been performed.

- The quantities {vfhA, tifA}, {vftA, afa}, {vhmA, hmA, thmA}, {vtmA, htA, ttmA} are assigned now the corresponding values given in the printout.

To obtain the numerical values of the those quantities just enter the name of the quantity into an input cell and press SHIFT - ENTER.

**vfhA**

7.92893

**tifA**

2.96704

**vftA**

27.2482

**afa**

73.0828

**vhmA**

7.92893

**hmA**

0.000439053

**thmA**

2.96702

**vtmA**

27.2482

**htA**

$2.27111 \times 10^{-11}$

**ttmA**

2.96704

- The functions {xhA, xzA, vhA, vzA, vtA, atA} can be evaluated at any point between 0 and tifA.

To find the numerical value **func[t]**, where **func** can be any of the functions above and **t** a point in time between 0 and tifA enter the command **func[t]** into an input cell and press SHIFT - ENTER. For example,

```
xhA[1.2]
```

```
2.42016
```

```
xvA[1.2]
```

```
33.1658
```

```
vhA[1.2]
```

```
3.89691
```

```
vzA[1.2]
```

```
11.2116
```

```
vtA[1.2]
```

```
11.8695
```

```
atA[1.2]
```

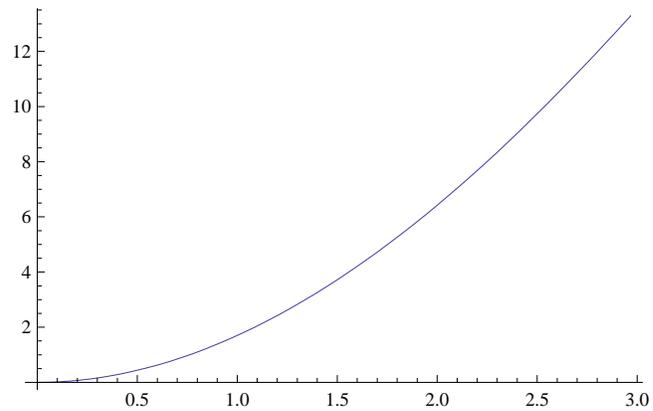
```
19.1663
```

- These functions can also be plotted on the interval  $[0, \text{tifA}]$ .

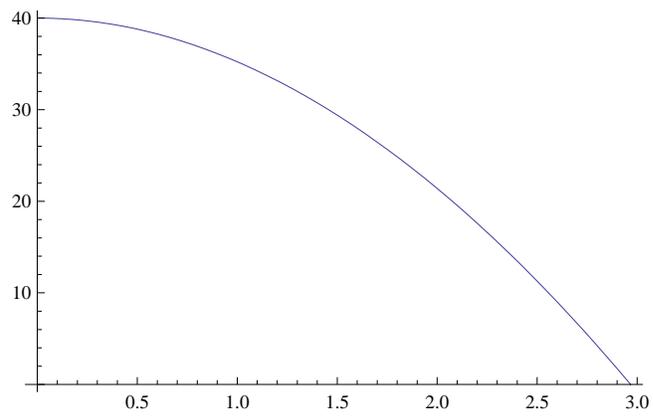
To graph a function **func**, where **func** can be any of the functions above, enter the command `Plot[func[t], {t, 0, tifA}]` into an input cell and press SHIFT-ENTER.

For example,

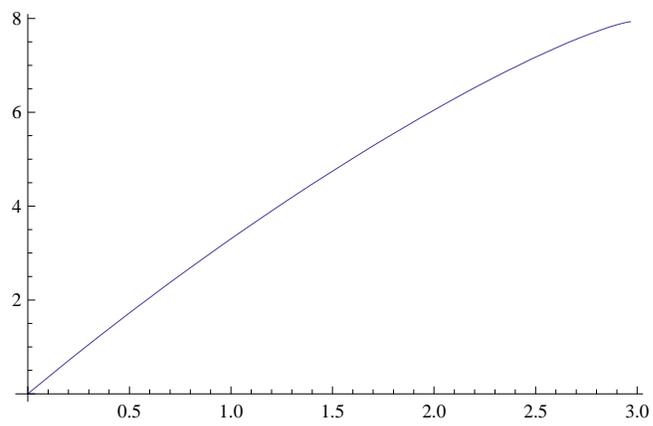
```
Plot[xhA[t], {t, 0, tifA}]
```



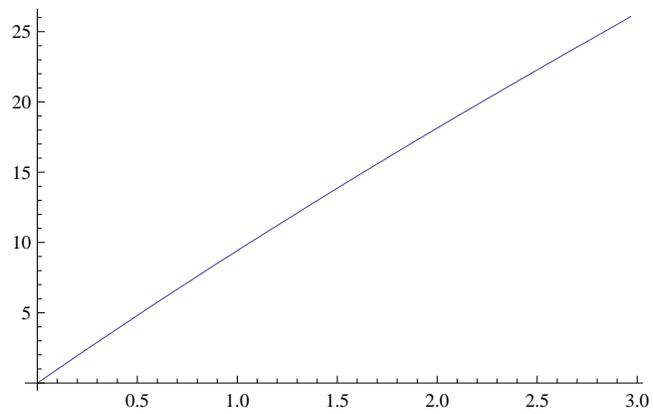
```
Plot[xvA[t], {t, 0, tifa}]
```



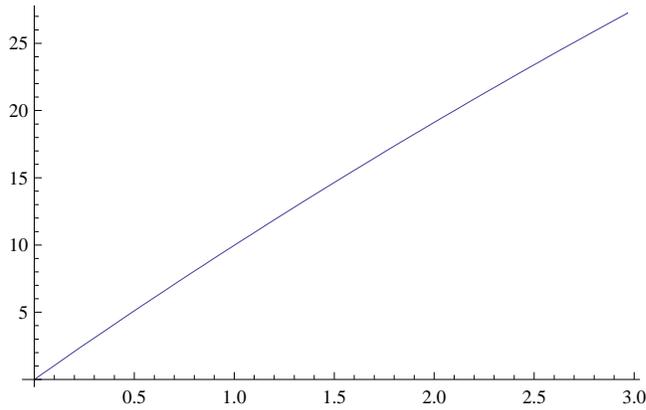
```
Plot[vhA[t], {t, 0, tifa}]
```



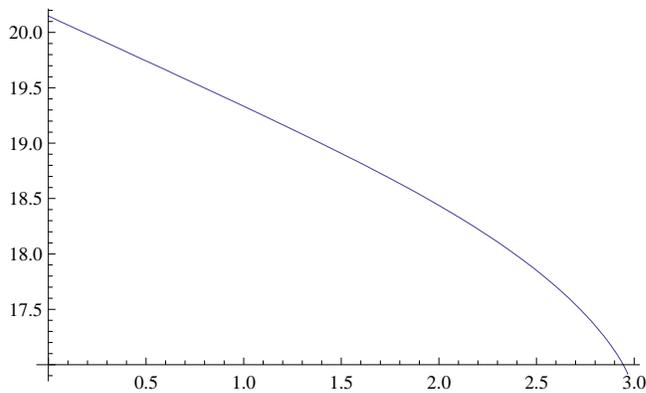
```
Plot[vzA[t], {t, 0, tifa}]
```



Plot[vtA[t], {t, 0, tifa}]



Plot[atA[t], {t, 0, tifa}]



- 3.2.2 We can repeat the same commands for the second example considered in subsection 2.3. Note the differences between the two examples.

```
{ {vfhA, tifa}, {vftA, afA}, {vhmA, hmA, thmA}, {vtmA, htA, ttmA},
  {xhA, xvA, vhA, vzA, vtA, atA} = missileVelocitiesA[0.0885`, 2, 150, 30];
```

a = 0.0885, terrain = 2, V10 = 150 m/s, h = 30 m

terminal horizontal velocity = 101.415 m/s

terminal total velocity = 101.584 m/s

angle of impact with ground = 3.30295 degrees

maximum horizontal velocity = 141.069 m/s attained at a height of 20.4574 m

maximum total velocity = 141.359 m/s attained at a height of 20.252 m

- The quantities {vfhA, tifa}, {vftA, afA}, {vhmA, hmA, thmA}, {vtmA, htA, ttmA} are assigned now the corresponding values given in the printout.

**vfhA**

101.415

**tifa**

4.21472

**vftA**

101.584

**afA**

3.30295

**vhmA**

141.069

**hmA**

20.4574

**thmA**

1.92754

**vtmA**

141.359

**htA**

20.252

**ttmA**

1.95025

- The functions {xhA, xzA, vhA, vzA, vtA, atA} can be evaluated at any point between 0 and tifi. For example,

**xhA[1.2]**

146.48

**xvA[1.2]**

26.0957

**vhA[1.2]**

139.188

**vzA[1.2]**

6.22307

**vtA[1.2]**

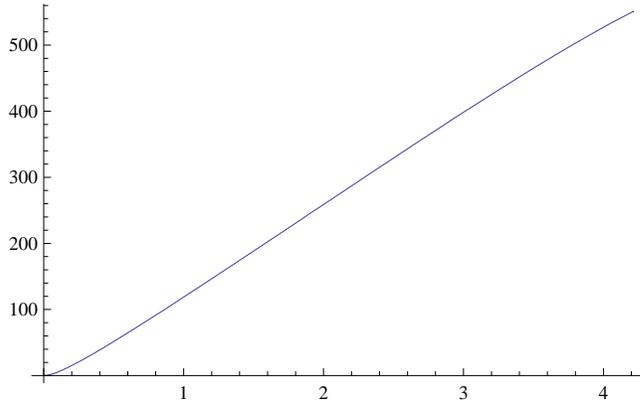
139.327

**atA[1.2]**

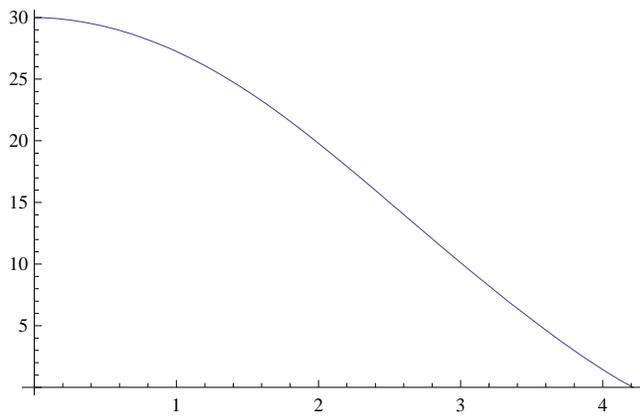
87.44

- These function can also be plotted on the interval  $[0, t_{ifA}]$ :

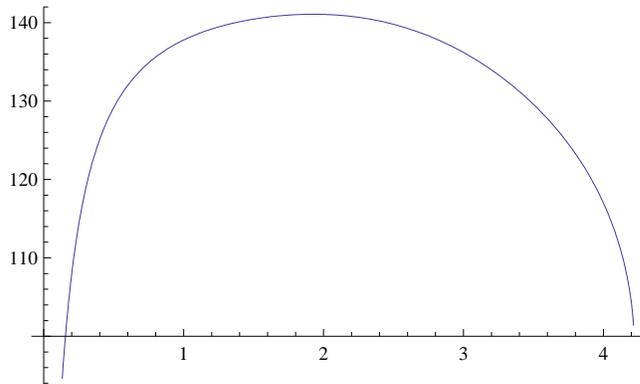
Plot[xhA[t], {t, 0, t<sub>ifA</sub>}]



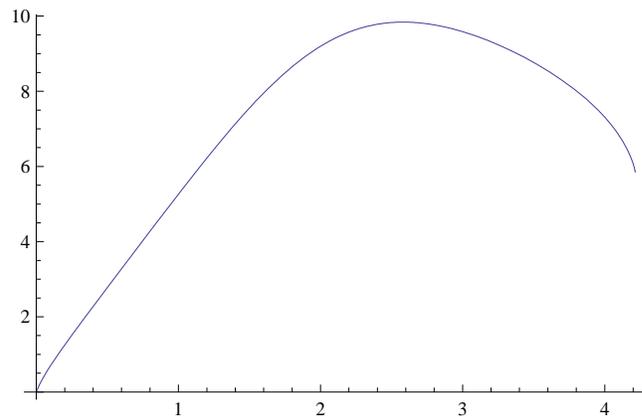
Plot[xvA[t], {t, 0, t<sub>ifA</sub>}]



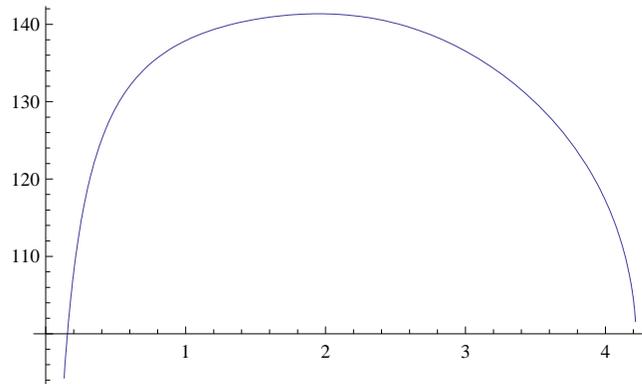
Plot[vhA[t], {t, 0, t<sub>ifA</sub>}]



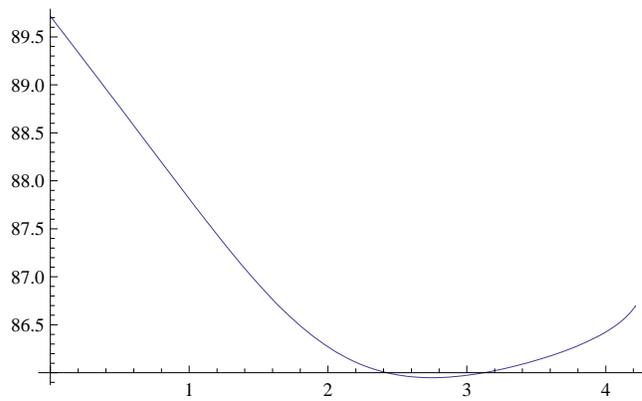
```
Plot[vzA[t], {t, 0, tifa}]
```



```
Plot[vtA[t], {t, 0, tifa}]
```



```
Plot[atA[t], {t, 0, tifa}]
```



#### 4. Installing and using the examples.nb notebook

The examples presented in this software manual can be found in the *Mathematica* notebook **examples.nb**. In order to be able to use this notebook proceed as follows.

4.1. If you have not already done so, install the subroutines contained in **mv.nb** by following steps 1.1-1.5 from section 1.

4.2. Save a copy of the file **examples.nb** in the current directory.

4.3. Click on the icon of the file **examples.nb**.

4.4. Initialize the input cells one by one by highlighting the cell's bracket (or setting the cursor on the same line as the command) and pressing SHIFT-ENTER.

You may then create your own examples at the end of the notebook, or in a new *Mathematica* notebook.