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NOTES AND COMMENTS

Determining Longitudinal Integral Turbulence Scales in the Near-Neutral Atmospheric Surface Layer

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Abstract
We briefly assess approaches used to date for the estimation of the longitudinal integral turbulence scale L_u^x in the near-neutral atmospheric surface layer, and propose an approach based on recent theory and measurements. A closed-form expression is derived according to which L_u^x is proportional to the height z above the surface. The factor of proportionality depends upon two non-dimensional parameters: the measured lowest Monin frequency f_s for which the non-dimensional spectrum conforms to Kolmogorov s two-thirds law, and the ratio $\beta = \overline{u^2}/u_*^2$, where $\overline{u^2}$ and u_* denote the mean square value of the longitudinal velocity fluctuations and the friction velocity, respectively.

Keywords Atmospheric surface layer · Integral length scale · Monin frequency · Neutral stratification · Turbulence spectra.

1 Introduction

The longitudinal integral scale of turbulence in the near-neutral atmospheric surface layer (ASL), denoted by L_u^x, is associated with the energy-containing turbulent eddies and plays an important role in a wide variety of applications. In particular, it affects the flow-induced aerodynamic load on engineering structures (Ho et al. 2003). An effective approach to determining L_u^x is therefore needed. However, such an approach has not been available to date owing to difficulties that prompted Panofsky and Dutton (1984, p. 176) to "recommend that integral scales be avoided in applications to atmospheric data. Many investigators have com-

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puted integral scales from atmospheric data, but the results are badly scattered and cannot be organized." Some researchers have proposed an approach based on the frequency n_{max} for which the curve $nS_u(n)$ is a maximum (n is the frequency, $S_u(n)$ is the spectral density of the longitudinal velocity fluctuations). "Unfortunately, the curves $nS_u(n)$ tend to be quite flat and sufficiently variable that n_{max} is not well-defined" (Panofsky and Dutton 1984). Indeed, in the ASL, the large-scale eddies are more energetic, obscuring the peak. Pasquill and Butler (1964) also warn against the use of this approach, and note that it likely underestimates L_u^x by a factor of two or three.

An alternative approach to determining L_u^x is to make use of its dependence upon the spectrum $S_u(n)$. Available models of the spectrum were until recently insufficiently developed to allow credible results to be obtained by this approach. However, theoretical, numerical and experimental results obtained by, e.g., Tchen (1953), Tchen (1954), Richards et al. (1997), Lauren et al. (1999), Hunt and Carlotti (2001), Carlotti (2002), Högström et al. (2002) and confirmed by measurements reported by Drobinsky et al. (2004), were used to develop an expression for the spectrum $S_u(n)$ that is satisfactory for engineering purposes, to within approximations inherent in the following: (i) the Taylor hypothesis, (ii) the assumption, well accepted for structural engineering purposes, that a spectral gap exists at frequencies of the order of 1/3600 Hz that separates mesometeorological from micrometeorological flow fluctuations, and (iii) the use in practical applications of data samples approximately representative of stationary processes. As noted by an anonymous reviewer, the common assumption that the spectral gap occurs at frequencies of the order of 1/3600 Hz may in fact not be appropriate. The practical consequences of deviations from that assumption should in our opinion be the subject of future research.

The purpose of this Note is to use the relation between L_u^x and $S_u(n)$ to derive a closed-form expression that yields the integral scale L_u^x as a function of height above the surface, z, the ratio $\beta = \overline{u^2}/u_*^2$ (where $\overline{u(z_0, t)^2}$ is the mean square value of the longitudinal velocity fluctuations, and u_* is the friction velocity obtained from the logarithmic law $U(z, z_0) = (u_*/k) \ln(z/z_0)$), and Monin frequency f_s, where f_s is the lower limit of the inertial subrange.

In the following, Sect. 2 is devoted to a critique of spectral models proposed by Davenport (1961), Kaimal et al. (1972) and the ASCE/SEI (2012), and explains why these models could not be used to obtain adequate information on L_u^x. Section 3 presents an expression for the spectrum $S_u(n)$ consistent with recent research results, and the derivation of the closed-form expression for L_u^x consistent with that spectrum. Section 4 presents conclusions.

2 Early Expressions for $S_u(z, n)$ and their Relation to the Integral Turbulence Scale L_u^x

We consider the extent to which early expressions for the spectra $S_u(z, n)$ are adequate for the purpose of determining integral longitudinal turbulence scales by using the well-known relation

$$S_u(z, z_0, n = 0) = \frac{4\beta(z_0)u_*^2 L_u^x(z, z_0)}{U(z)} \tag{1}$$

where, within the near-surface sublayer, the nominal values of the factor $\beta(z_0)$ are approximately 6.25 over water ($z_0 \approx 0.005$ m), 6.0 over open terrain ($z_0 \approx 0.03$ m), 5.25 over suburban terrain ($z_0 \approx 0.3$ m), and 4.85 over towns or woodland ($z_0 \approx 1.0$ m) (Biétry et al. 1978; Stull 2015). Equation 1 is a form of the relationship between the correlation function and the spectrum.

2.1 Davenport Spectrum

The following expression for the spectrum of the longitudinal turbulent fluctuations in the neutrally-stratified ASL was proposed by Davenport (1961),

$$\frac{nS_u(n)}{u_*^2} = 4.0\frac{x^2}{\left(1+x^2\right)^{4/3}} \tag{2}$$

in which $x = 1200n/U_{10}$, the frequency n is in Hz, U_{10} is the mean wind speed in m s^{-1} at elevation $z = 10$ m. The area under the spectral curve given by Eq. 2 is $6u_*^2$ for any roughness length z_0. Equation 2 implies that $S_u(n)$ does not depend upon z, and that $S_u(n)|_{n=0} = 0$, implying that inherent in Eq. 2 is an integral scale $L_u^x = 0$.

2.2 Kaimal Spectrum

The following expression for the longitudinal velocity spectrum was proposed by Kaimal et al. (1972),

$$\frac{nS_u(z,n)}{u_*^2} = \frac{105f}{(1+33f)^{5/3}} \tag{3}$$

where $f = nz/U(z)$. In the inertial subrange Eq. 3 is for practical purposes equivalent to the well-accepted Monin expression for the spectrum in the ASL

$$\frac{nS_u(z,n)}{u_*^2} = 0.26f^{-2/3}, \tag{4}$$

while the spectral ordinate at $n = f = 0$ corresponding to Eq. 3 is

$$S_u(z, n=0) = \frac{105u_*^2 z}{U(z)}. \tag{5}$$

For Eq. 5 to conform to Eq. 1 it is necessary to have

$$L_u^x(z) = \left(\frac{105}{4\beta}\right) z. \tag{6}$$

According to Eq. 6, for the values of β listed earlier, $L_u^x(z)$ increases as z_0 increases. Available data suggest, however, that $L_u^x(z)$ decrease significantly as the roughness length z_0 increases. For example, the ASCE 49-12 Standard (ASCE/SEI 2012) lists values of L_u^x at 10 m above the surface at mid-latitudes as functions of surface roughness length as follows: 190 m ($z_0 = 0.0002$ m), 140 m ($z_0 = 0.0005$ m), 110 m ($z_0 = 0.03$ m), 64 m ($z_0 = 0.25$ m), and 45 m ($z_0 = 1$ m). The decrease of $L_u^x(z)$ as z_0 increases is also noted by Counihan (1975). Finally, for all values of z_0, the area under the spectral curve implicit in Eq. 4 corresponds to $\beta = 4.77$, which may result in the underestimation of the flow-induced aerodynamic forces acting on structures.

2.3 Von Kármán Spectrum

We now consider the expression

$$\frac{nS_u(z,n)}{u_*^2} = \frac{4\beta \frac{nL_u^x}{U(z)}}{\left(1 + 70.8\left(\frac{nL_u^x}{U(z)}\right)^2\right)^{5/6}}. \tag{7}$$

Equation 7 was adopted by ASCE/SEI (2012), which states that it "was developed by von Kármán (1948) based on the theory of isotropic turbulence", rather than for the description of anisotropic turbulent flows that characterize the near-neutral ASL. Equation 7 was originally specified for use in a flight-safety context, with L_u^x = 760 m for medium to high altitudes where the turbulence does indeed tend to be isotropic. For details on Eq. 7, see Harris (1990).

For Eq. 7 to conform in the inertial subrange to the widely accepted Eq. 4 it would be necessary that

$$L_u^x(z, z_0) = 0.29\beta^{3/2}z, \tag{8}$$

and for z = 10 m, z_0 = 0.03 m, and β = 6.0, Eq. 8 yields L_u^x = 41 m, whereas the value listed in ASCE/SEI (2012) is L_u^x = 110 m.

3 Expressions for $S_u(z, z_0, n)$ and $L_u^x(z, z_0)$ Based on Drobinsky et al. (2004)

The spectrum $S_u(z,n)$ based on Drobinsky et al. (2004), (i) satisfies Eq. 4 in the inertial subrange (i.e., for frequencies $n \geq n_s$, where n_s is the lower limit of the intertial subrange), (ii) varies as $a(z,z_0)/n$ in the interval $n_l \leq n < n_s$, where the frequency n_l remains to be determined, and (iii) is equal to $c(z,z_0)$ in the interval $0 < n < n_l$ (see Fig. 1). It is required, in addition, that (iv) the spectral curve $S_u(z,n)$ be continuous at $n = n_l$ and $n = n_s$, (v) the area under the spectral curve $S_u(z,n)$ be equal to $\overline{u(z, z_0, t)^2}$, and (vi) $\mathrm{d}S_u/\mathrm{d}n|_{n=0} = 0$. The coefficients $a(z,z_0)$, $c(z,z_0)$ and the frequency n_l can be determined from conditions (iv) and (v). The frequency n_s is obtained by measurement (Drobinsky et al. 2004), thus,

$$S_u(z, z_0, n) \begin{cases} \dfrac{a(z,z_0)}{n_l} & 0 < n < n_l \qquad (9) \\[2ex] \dfrac{a(z,z_0)}{n} & n_l < n < n_s \qquad (10) \\[2ex] 0.26u_*^2\left(\dfrac{z}{U(z,z_0)}\right)^{-2/3} n^{-5/3}. & n_s \leq n \qquad (11) \end{cases}$$

As noted by Katul and Chu (1998), Eq. 10 was originally proposed by Tchen (1953), see also Banerjee and Katul (2013), Banerjee et al. (2015) and Banerjee et al. (2016).

Using the notation

$$\frac{n_s z}{U(z)} = f_s \tag{12}$$

for $n = n_s$, Eq. 11 becomes

$$S_u(z, z_0, n_s) = 0.26u_*^2 f_s^{-5/3}\frac{z}{U(z,z_0)}, \tag{13}$$

and the condition that the functions defined by Eqs. 10 and 11 are continuous at $n = n_s$ yields

$$a(z, z_0) = 0.26u_*^2 f_s^{-2/3}. \tag{14}$$

A plot of $nS_u(z, z_0, n)/u_*^2$ corresponding to z = 10 m, z_0 = 0.04 m, and U_{10} = 5.39 m s^{-1} is shown in Fig. 1.

The area under the spectral curve in the intervals $0 \le n \le n_l$ is $[a(z, z_0)/n_l]\, n_l = a(z, z_0)$, and the areas under the spectral curve in the intervals $n_l \le n_s$ and $n \ge n_s$ are given, respectively, by

$$\int_{n_l}^{n_s} 0.26u_*^2 f_s^{-2/3} \frac{\mathrm{d}n}{n} = 0.26u_*^2 f_s^{-2/3} \ln \frac{n_s}{n_l}, \tag{15}$$

$$\int_{n_s}^{n_d} 0.26u_*^2 \left(\frac{z}{U(z)}\right)^{-2/3} n^{-5/3}\mathrm{d}n \approx 0.39u_*^2 f_s^{-2/3}, \tag{16}$$

where n_d is the frequency corresponding to the onset of dissipation by molecular processes and may for practical purposes be assumed in Eq. 16 to be infinite.

The total area under the spectral curve is

$$\beta(z_0)\, u_*^2 = 0.26u_*^2 f_s^{-2/3} + 0.26u_*^2 f_s^{-2/3} \ln \frac{n_s}{n_l} + 0.39u_*^2 f_s^{-2/3}, \tag{17}$$

which yields

$$\begin{aligned} n_l &= n_s \exp\left[-\frac{\beta(z_0) - 0.26 f_s^{-2/3} - 0.39 f_s^{-2/3}}{0.26 f_s^{-2/3}}\right] \\ &= f_s \frac{U(z)}{z} \exp\left[-\frac{\beta(z_0) - 0.65 f_s^{-2/3}}{0.26 f_s^{-2/3}}\right]. \end{aligned} \tag{18}$$

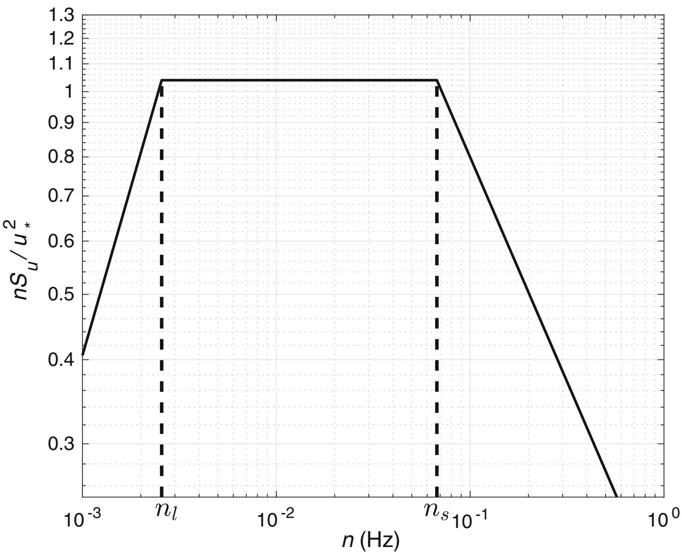

Fig. 1 Plot of non-dimensional spectrum $nS_u(z, z_0, n)/u_*^2$ (z = 10 m, z_0 = 0.04 m, U_{10} = 5.39 m s^{-1})

It follows from Eqs. 1, 9, 14 and 18 that the longitudinal integral turbulence scale is

$$L_u^x(z, z_0) = \frac{S_u(n=0, z, z_0)U(z, z_0)}{4\beta(z_0)u_*^2} = \left[\frac{0.26 f_s^{-5/3}}{4\beta(z_0)} \exp \frac{\beta(z_0) - 0.65 f_s^{-2/3}}{0.26 f_s^{-2/3}}\right] z \quad (19)$$

Consider the following example: let z = 10 m, U_{10} = 5.39 m s^{-1}, z_0 = 0.04 m. Therefore $\beta(10\text{ m}) \approx 6$ and u_* = 0.39 m s^{-1}. According to measurements reported in Drobinsky et al. (2004), f_s = 0.125 for all elevations from 1.5 m to 55 m. Then n_s = 0.0674 Hz (Eq. 12), n_l = 2.56×10^{-3} Hz (Eq. 18), a(10 m) = 0.158 (Eq. 14), $S_u(n_l$, 10 m) = 61.7 m^2 s^{-1} = $S_u(n = 0$, 10 m) (Eqs. 9 and 10). The non-dimensional spectral density is plotted in Fig. 1, and the calculated integral scale is $L_u^x = 9.11z$ = 91.1 m (Eq. 19).

The measurements of Drobinsky et al. (2004) have consistently yielded the value f_s = 0.125 at all six elevations for which data were obtained. It can be verified that for values of f_s equal to 0.9 × 0.125 and 1.1 × 0.125, the corresponding estimated values of L_u^x are 73.5 m and 113.5 m respectively. The finding that the curve $nS_u(n)$ is flat in the range $n_l < n < n_s$ confirms the statement in Panofsky and Dutton (1984) and Pasquill and Butler (1964) that the frequency for which that curve attains a maximum yields no useful information on the integral length.

4 Conclusions

The longitudinal integral scale L_u^x of turbulence in the near-neutral atmospheric surface layer is associated with the energy-containing turbulent eddies that affect the flow-induced aerodynamic loads on engineering structures. This Note has developed a closed-form expression for L_u^x (Eq. 19), according to which L_u^x is proportional to the height z above the surface and depends upon, (i) the square of the ratio between the r.m.s. value of longitudinal velocity fluctuations and the friction velocity, and (ii) the lowest Monin frequency f_s for which the spectrum conforms to Kolmogorov's 2/3 law. The value f_s = 0.125 was obtained consistently at all six elevations for which data were obtained by Drobinsky et al. (2004).

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