Optimization and Multihazard Structural Design

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Abstract: There is a growing interest in the development of procedures for the design of structures exposed to multiple hazards. The goal is to achieve safer and/or more economical designs than would be the case if the structures were analyzed independently for each hazard and an envelope of the demands induced by each of the hazards were used for member sizing. We describe an optimization approach to multihazard design that achieves the greatest possible economy while satisfying specified safety-related and other constraints. We then present an application to illustrate our approach.

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Introduction

There is a growing interest in the development of procedures for the design of structures exposed to multiple hazards. The goal is to achieve safer and/or more economical designs than would be the case if the structures were designed independently for each of the hazards and an envelope of the demands induced by each hazard were used for member sizing.

Useful, if mostly ad hoc, approaches to multihazard design have recently been proposed (Bruneau 2007). However, a broad, multidisciplinary foundation for multihazard design remains to be developed. Such a foundation should include a probabilistic component. Duthinh and Simiu (2008) have found that the current ASCE 7 Standard (ASCE 2005) does not take into account the fact that failure probabilities of structures in regions exposed to both strong wind hazard and strong earthquake hazard may exceed their counterparts in regions exposed to only one of the hazards, and that for this reason consideration should be given to augmenting wind and seismic load factors specified in the ASCE 7 Standard so that risk-consistency be achieved within the Standard. The mathematically and physically rigorous rationale advanced by Duthinh and Simiu (2008) in support of these statements can be conveyed intuitively by noting that health and life insurance premiums would likely be higher for a professional motorcycle racer if he/she would also be active as a stunt artist.

The purpose of this note is to submit that a multidisciplinary foundation of multihazard design theory would benefit from the inclusion of appropriate optimization approaches. For simplicity we address in this note the case in which the loads corresponding to a nominal probability of exceedance of the failure limit state are specified. We explore the potential of optimization under multiple hazards as a means of integrating the design so that the greatest possible economy is achieved while satisfying specified safety-related and other constraints.

Our formulation the multihazard design problem rests on the fact that optimization under N hazards $(N>1)$ imposes m_i $(i=1,2,\ldots,N)$ sets of constraints, all of which are applied simultaneously to the nonlinear programming problem (NLP) associated with the design. Following a description of our approach we present a simple illustrative application. Finally, we present a set of conclusions and suggestions for future research.

Multihazard Design as a Nonlinear Programming Problem

We consider a set of n variables (i.e., a vector $\mathbf{d}$ with n components $d_1,d_2,\ldots,d_n$) characterizing the structure. In a structural engineering context we refer to that vector as a *design*. Given a single hazard, we subject those variables to a set of *m constraints*

$$g_1(d_1,d_2,\ldots,d_n) \leqslant 0$$

$$g_2(d_1,d_2,\ldots,d_n) \leqslant 0,\ldots,g_m(d_1,d_2,\ldots,d_n) \leqslant 0 \qquad (1)$$

Examples of constraints are minimum or maximum member dimensions, allowable stresses or design strengths, allowable drift, allowable accelerations, and so forth. A design $(d_1,d_2,\ldots,d_n)$ that satisfies the set of m constraints is called *feasible*. Optimization of the structure consists of selecting, from the set of all feasible designs, the design denoted by $(\bar{d}_1,\bar{d}_2,\ldots,\bar{d}_n)$ that minimizes a specified *objective function* $f(d_1,d_2,\ldots,d_n)$. The objective function may represent, for example, the weight or cost of the structure. The selection of the optimal design is a NLP for the solution of which a variety of techniques are available. We emphasize that in this note we do not consider topological optimization. Rather, we limit ourselves to structures whose configuration is specified, and whose design variables consist of member sizes.

As noted earlier, in multihazard design each hazard i $(i=1,2,\ldots,N)$ imposes a set of m_i constraints. Typically the optimal design under hazard i is not feasible under (i.e., does not satisfy the constraints imposed by) hazard $j\neq i$. For example,

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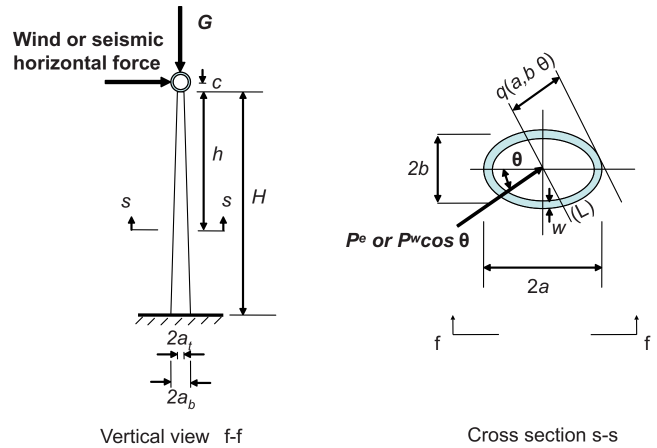

Fig. 1. Vertical view and cross section of column

the optimal design that satisfies the drift constraints under one hazard may not satisfy drift constraints (i.e., may not be feasible) under another hazard. Common engineering practice is to obtain separately feasible designs d^i corresponding to each hazard i ($i = 1, 2, \ldots, N$). Those designs are used to construct an envelope d such that the constraints imposed under all hazards are satisfied. However, such a design will in general be suboptimal.

The NLP is clearly more difficult to solve in the multihazard than in the single hazard case. However, progress made during the last two decades in the field of nonlinear programming (see, e.g., Wright 2005) now renders the solution of complex multihazard problems a practical possibility.

Application

The purpose of the application in this section is to illustrate the potential for using NLP to solve multihazard structural optimizations problem. A thorough and detailed analysis and design, global and local buckling, stochastic loading, dynamic response effects and constraints, and topological optimization are outside the scope of this exercise, and will be addressed in future phases of this project.

Description of Structure

We consider the structure of Fig. 1, representing one in a long row of columns free at the top and fixed at the base that support a pipe filled with water. The water is heated by ground-level computer-controlled rotating mirrors that focus reflected solar rays onto the pipe. The resulting heat is used for power generation. For the sake of simplicity and clarity in the illustration of our approach we assume that, in response to functionality requirements, the supporting structure consists solely of the columns (i.e., no bracing or other members are present). The column is subjected to a gravity load G due to the weight of the pipe and the water it contains, and to the wind load P^w and seismic load P^e acting nonsimultaneously. The seismic load can act from any direction. The wind load, however, is a function of the wind velocity direction. Its magnitude is $P^w = P_0^w \cos\theta$, where θ is the angle between the direction of the wind velocity vector and the normal to the longitudinal axis of the pipe. The horizontal cross section of the column is tubular and elliptical in shape, with constant thickness w. The major axis of the outer ellipse, $2a(h)$, at a cross section with coordinate h (see Fig. 1) varies linearly between $2a_t$ at the top of the column and $2a_b$ at the base; its direction coincides with the normal to the longitudinal axis of the pipe. The minor axis of the outer ellipse, $2b(h)$, at a cross section with coordinate h varies linearly between $2b_t$ at the top of the column and $2b_b$ at the base. Had the column been exposed to seismic hazard alone, a circular cross sectional shape would have been warranted. However, because the wind force is largest when its direction is normal to the longitudinal axis of the pipe, the section modulus of the cross section needs to be larger in that direction. One interesting output of the procedure is the ratio a/b corresponding to the optimal multihazard design. In the absence of earthquakes, that ratio should be as large as possible. In the absence of wind, the ratio should be unity. For the multihazard design, an intermediate ratio will be appropriate. That ratio will be neatly yielded by the optimization procedure.

The semimajor and semiminor axes of the ellipse at coordinate h are

$$a(h) = \left(1 - \frac{h}{H}\right)a_t + \frac{h a_t b_b}{H b_t}, \quad b(h) = \left(1 - \frac{h}{H}\right)b_t + \frac{h}{H}b_b \tag{2}$$

where H=height of the column. Henceforth we denote $a(h)$ and $b(h)$ by a and b, respectively.

Stresses

The compression stresses due to *gravity* loads (i.e., the sum of the force G and of the column weight from the top to the cross-section with coordinate h, divided by the area $A(h)$ of the column at that cross-section) is

$$\sigma_g(\mathbf{d}, h, G) = \frac{G}{A(h)} + \frac{\gamma \int_0^h A(z)dz}{A(h)} \tag{3a}$$

$$A(z) = A(z) = \pi\{a(z)b(z) - [a(z) - w]\{\ b(z) - w]\}\} \tag{3b}$$

$$\sigma_g(\mathbf{d}, h, G) = \frac{b_t HG + \pi\gamma b_t H w h(a_t + b_t - w) + \pi w \gamma h^2 (a_t + b_t)(b_b - b_t)/2}{\pi w[H(a_t + b_t - w)b_t + (a_t + b_t)(b_b - b_t)h]} \tag{3c}$$

where $\mathbf{d}$=vector (a_t, b_t, a_b, b_b, w) (note that, for given a_t, b_t, b_b, the dimension a_b is determined); H=height of the column; w=thickness of the tubular column (Fig. 1); and γ=specific weight of the column material.

The maximum compression stresses due to *bending* induced by a horizontal force P acting at elevation $H+c$ above the column base in the direction θ with respect to the major axis of the column cross section is

$$\sigma_g(\mathbf{d}, h, P, \theta) = \frac{P(c+h)q(a,b,\theta)}{I(a,b,w,\theta)} \tag{4}$$

The distance $q(a,b,\theta)$ and the moment of inertia $I(a,b,w,\theta)$ about the axis (D) (see Fig. 1) can be shown to be

$$q_0[\ a,b,\theta)] = \{0.5[a^2 + b^2 + (b^2 - a^2)\cos(2\theta)\ \}^{1/2} \tag{5}$$

$$I[a,b,w,\theta] = \frac{\pi}{8}\{ab[a^2 + b^2 + (b^2 - a^2)] - (a-w)(b-w)\{(a-w)^2 + (b-w)^2 + [(b-w)^2 - (a-w)^2]\}\}\cos(2\theta) \tag{6}$$

Table 1. Multihazard Optimization[a]

$w_{\min}$	b_t	a_t	b_b	w	Weight	msW	hW	msQ	hQ	iter
5.08	40.64	63.98	144.8	5.08	167.24	110.3	1,834	110.3	1,793	7
3.81	46.16	73.66	167.3	3.81	145.54	110.3	1,834	110.3	1,806	9
2.54	56.46	91.23	206.1	2.54	124.20	110.3	1,839	110.3	1,821	8

[a]Dimensions: mm; weight: N; and stresses: MPa.

Multihazard Optimization

We wish to find the design variables a_t, b_t, b_b, w so that the largest compression stress in the column not exceed the allowable stress σ_{all}. The design variables are subjected to the constraints

$$w_{\min} \leqslant w \leqslant b_{t,\min} = 38.1 \text{ mm}(1.5 \text{ in.}) \leqslant b_t \leqslant a_t \leqslant a_b \leqslant a_{b,\max} = 228.6 \text{ mm}(9 \text{ in.}) \quad (7)$$

where $w_{\min}$ and $b_{t,\min}$=lower bounds of w and b_t, respectively, and $a_{b,\max}$=upper bound of a_b. The largest compression stresses in the column under earthquake and wind loading are, respectively

$$\sigma^e_{mh}(\mathbf{d},h) = \sigma_g(\mathbf{d},h) + [\sigma_b(\mathbf{d},h,P^e,\theta = \pi/2)] \leqslant \sigma_{\text{all}}, \quad 0 \leqslant h \leqslant H \quad (8)$$

$$\sigma^w_{mh}(\mathbf{d},h) = \sigma_g(\mathbf{d},h) + \max_{\theta \in [0,\pi]} [\sigma_b(\mathbf{d},P^w,\theta)] \leqslant \sigma_{\text{all}}, \quad 0 \leqslant h \leqslant H \quad (9)$$

Note that the ratio between the semiaxes of the ellipse could be used as a parameter in the computation. However, this would render the problem more strongly nonlinear and make it more difficult to solve. Denoting

$$\bar{g}^w_{mh}(\mathbf{d},h) = \sigma^w_{mh}(\mathbf{d},h) - \sigma_{\text{all}} \quad (10)$$

$$\bar{g}^e_{mh}(\mathbf{d},h) = \sigma^e_{mh}(\mathbf{d},h) - \sigma_{\text{all}} \quad (11)$$

the multihazard optimization problem becomes

$$\min_{a_t,b_t,b_b,w} f(\mathbf{d},h) \quad (12)$$

such that

$$\bar{g}^w_{mh}(\mathbf{d},h) \leqslant 0$$

$$\bar{g}^e_{mh}(\mathbf{d},h) \leqslant 0$$

$$w_{\min} \leqslant w \leqslant b_{t,\min} = 38.1 \text{ mm} \leqslant b_t \leqslant a_t \leqslant a_b \leqslant a_{b,\max} = 228.6 \text{ mm}$$

where the objective function f=weight of the column. The optimization problem Eq. (12) is in general considerably more difficult to solve than its counterparts where each hazard is considered separately. However, its solution can be obtained efficiently by using modern optimization techniques.

Optimization Algorithm

The inequalities $\bar{g}^w_{mh}(\mathbf{d},h) \leqslant 0$ and $\bar{g}^e_{mh}(\mathbf{d},h) \leqslant 0$ in Eq. (12) impose infinitely many constraints on the design variables a_t, b_t, b_b, and w (i.e., they impose a constraint for each value of h). For this reason, we replace these constraints by a finite number of constraints of the type

$$\bar{g}^w_{mh}(\mathbf{d},h_i) \leqslant 0 \quad i = 1,2,\ldots,k \quad (13)$$

where $0 \leqslant h_1 \leqslant h_2 \leqslant \ldots \leqslant h_k \leqslant H$=finite partition of the interval $[0,H]$. The set of coordinates h_i, denoted by $\boldsymbol{H}$, is obtained as follows. Initially the set consists of four points: 0, $H/2$, H, and rand$_1$, where rand$_1$ is a random number in the interval $[0,H]$.

We replace in Eqs. (10) and (11) the allowable stress σ_{all} by $\sigma_{\text{all}} - \varepsilon/(2\pi w)$. (In our calculations we chose ε=0.1.) We then compute the maximum stresses under the wind hazard and under the earthquake hazard, msW and msQ, respectively, and, to within a tolerance of 0.01ε, the coordinates $h=hW$ and $h=hQ$ at which those stresses occur. We return the set (a_t,b_t,b_b,w) as the solution of the optimal design problem if the stresses msW and msQ are both smaller than $\sigma_{\text{all}} - 0.01\varepsilon$. If this is not the case we update the set $\boldsymbol{H}$ by adding to it the point hW if $msW > \sigma_{\text{all}} - 0.01\varepsilon$, the point hQ if $msQ > \sigma_{\text{all}} - 0.01\varepsilon$, and the point with coordinate rand$_2$, and repeat the procedure until the requisite tolerance in the stresses msW and msQ is achieved.

Numerical Results

We assumed the following values of the constants defining our case study: H=3,660 mm, c=30.5 mm, vertical applied load G=31,150 N, P^e=8,010 N, P^w=8,010 N, γ=76,850 N/m^3. We have considered for a_b the upper bound $a_{b,\max}$=22.86 mm. For w the lower bound, chosen for reasons of constructability, is $w_{\min}$=5.08 mm. However, we consider lower values (see Table 1) to examine the theoretical effect of lowering $w_{\min}$. For b_t the lower bound is chosen to be $b_{t,\min}$=38.1 mm. We assumed that the column is constructed of A36 steel with allowable stress σ_{all} =110.3 MPa (24 ksi).

Table 1 lists the quantities a_t, b_t, b_b, w, msW, hW, msQ, and hQ, as well as the number of iterations *iter* required to achieve the specified tolerance. Note that in all cases w is equal to $w_{\min}$. This is due to the greater efficiency of increasing the larger axis of the ellipse, thereby achieving a larger section modulus than had the material been placed in a thicker member closer to the center of the ellipse. If a constraint had imposed a sufficiently small upper limit on that axis, the result $w > w_{\min}$ could have been obtained in some cases. For an actual design the dimensions shown in the table would have to be slightly modified for constructibility. The resulting weights and stresses would be modified accordingly.

Conclusions

In this note we presented an optimization-based approach to the design of structures in a multihazard environment. The advantage of an optimization approach in a multihazard context is that it provides an integrative framework allowing the structure to be optimized under constraints associated with all the hazards to which the structure is exposed, thereby achieving the most economical design consistent with the constraints of other types (e.g., constructability constraints) imposed on the structure. We applied such an approach to the particular case in which the hazards consist of earthquakes and winds. We illustrated our approach by considering a deliberately simple example, and did not make explicit allowance for local or global buckling, stochastic loading, dynamic effects, or probability considerations. However, for any specified topological configuration, the basic approach presented in this note is of general applicability. It could be applied, for example, to a high-rise frame structure for which optimization is carried out under a single hazard, as in Chan et al. (1995) and Spence and Simiu (2008), but under multiple hazards as well.

In general, multihazard optimization poses substantially greater challenges than optimization under a single hazard. However, recent progress in optimization theory and practice, and in particular the revolutionary development of interior point theory and algorithms, should allow the routine solution in the future of optimization problems under multiple hazards for structures of increasingly greater complexity. We view our note as a first step in that direction.

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